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TIME AND CROSS-SECTIONAL DIFFERENCES IN THE TAIL BEHAVIOR OF EURO INTEREST RATE FUTURE RETURNS

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Abstract

As response to the financial crisis in 2007/08 and the European sovereign debt crisis, the ECB started to conduct expansionary monetary policy on an unprecedented scale. In this paper I investigate the development of tail risks in the euro interest rate market since the implementation of this unconventional monetary policy. The focus of the study is on futures on German government bonds, namely the Bund, Bobl and Schatz, which are among the most relevant securities in this market. The analysis covers three aspects. First, I investigate if the daily returns of the futures exhibit fat tails over the period from 1999 to 2016 and if there are differences among these securities with respect to tail risk, as measured by the tail index. Second, I analyze if the tail risks are non-constant over the considered time period. Third, I study if the tail index contains information beyond the conventional risk measure volatility and its implications for value-at-risk considerations. Anticipating the results, this paper presents significant evidence for fat tails in the return distribution of the Bund, Bobl and Schatz future. In contrast to expectations, the results indicate the highest tail risk for the short-term Schatz future and the lowest for the long-term Bund future. Differences in market liquidity might be a reason for this. Furthermore, I find comprehensive evidence for an increase in right tail risk for all three futures around 2008. This increase is most significant for the long-term Bund future. Surprisingly, evidence for a decrease in left tail risk is found, although with lower significance. Additionally, the analysis reveals that tail index contains information, which is not captured by volatility. Thus, the results suggest that the accuracy of value-at-risk estimates for different long and short positions can be improved by taking into account the tail index explicitly in the estimation process.

Keywords: Extreme value theory, tail index estimation, optimal extreme sample fraction, structural change test, government bond futures, market risk management, monetary policy

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1 Introduction

Since the global financial crisis in 2008, risk management practices have become a major subject of debate in the financial industry. Due to the lack of regulation and proper risk management techniques prior to the crisis, global players like Lehman Brothers were able to perform highly questionable operations without estimating the underlying risk of these correctly. To make things worse, these risky operations were mainly financed with short-term funds, imposing further liquidity risk to the institution. After realizing severe losses due to the depreciation of mortgage-backed securities, rumours had spread in the market, questioning Lehman's solvency and refinancing possibilities. Eventually this business model had to come to an end. On September 15th 2008 Lehman Brothers declared bankruptcy, remaining the largest bankruptcy filing in the history of the United States. Consequently, markets reacted heavily to this event and global stock indices realized significant losses. This date is known as the high point of the most severe financial crisis since the Great Depression in 1930. However, the case of Lehman Brothers was not the first nor the last extreme event during the course of the crisis.

Due to the interconnection of financial markets, the crisis spread globally. Many institutions had similar toxic assets on their balance sheets and even direct exposure to Lehman Brothers. Mistrust within the financial markets grew, which resulted in a stop of lending within the banking system. Important reference interest rates like Euribor and Libor started to increase significantly. Overnight the European interbank market froze, imposing liquidity shortages on large and systemically relevant banks. To prevent a collapse of the system, the ECB had to intervene with unconventional monetary policy measures and provide unlimited liquidity to the financial markets, given adequate collateral. Additionally, European governments had to structure immense rescue packages for troubled institutions, costing billions of tax payers' money.

However, the economic consequences of this banking crisis remained fatal for many European countries. Not only were governments and the ECB challenged by economic contraction, but also by the European sovereign debt crisis, which persists until today. Especially peripheral countries like Portugal or Italy suffer from large amounts of public debt and household deficits relative to the size of their economies. Furthermore, the banking system in these countries is highly fragile, due to a large amount of non-performing loans and inadequately low capital buffers.[1] The potential need of the banks for further public assistance intensifies the crisis and encounters incomprehension by

[1] As of June 2015, non-performing loans account for 16.1% and 13.6% of total debt in Italy and Portugal, respectively. For comparison, the average within the euro area was 4.5% (Mesnard et al. 2016).

the citizens of these countries, who are confronted with poor economic conditions and reduced public benefits.

The heavy dependence of many European banks and even governments on the massive liquidity supply of the ECB questions the long-term sustainability of the euro system as a whole. The current policy conduct of the ECB resulted in a low interest rate environment, which leads to a distortion of economic incentives and increased risk taking of market participants. Consequently, the actions of the ECB and related market factors are closely monitored by the public. Releases of economic data, rating agency decisions or ECB press conferences have become key events for the European financial markets. These events are usually accompanied by heavy price fluctuations, as market participants try to anticipate the outcomes. Therefore, the questions arise if the likelihood of so-called extreme events has significantly increased since the financial crisis in 2008 and if the participants in the financial markets are now exposed to higher market risks, despite all regulatory efforts in the recent past. This stresses the importance of an accurate assessment and incorporation of extreme events in the market risk management of financial institutions.

Although great advancements in the assessment of extreme market events have been made since 2008, there is still large potential for further improvements and research in this area. Of special interest and significance is the euro interest rate market, which's function was heavily disrupted during the financial crisis and is currently distorted by a low interest rate environment. The rates for transactions worth trillions of euros are determined in this market. Not only is it important for the financial industry itself but also for the overall euro economy, since credit conditions for housing or business investment depend on it through the reference rates Euribor and Eonia. Additionally, the refinancing costs of the majority of European states, of which some face currently enormous financial constraints, are determined in this market.

Probably the most important and liquid securities in the euro interest rate market are futures on German government bonds. Since German government debt is considered as risk-free, it only reflects the general interest rate level in the euro zone and no credit conditions. Thus, futures on long-term German government bonds are used as indicator for future monetary policy and, even more importantly, as hedging instrument for the majority of euro interest rate transactions like swaps. Due to their monetary policy indicator function, these futures are especially effected by key events like releases of economic data or ECB press conferences about the future monetary policy conduct. The resulting tail risk in the return distributions questions their function as an effective hedging instrument for interest rate transactions or bond portfolios. However, it is naturally to assume that there exist differences between the futures' tail behavior, depending on the side of the distribution's tail and the maturity of the underlying German government bond. The aim of this paper is to identify time and cross-sectional differences in the tail behavior of these euro interest rates futures. In general, there are three different standardized future contracts on German government bonds. These are the so-called Bund, Bobl and Schatz future, which have a long-term, medium-term, and short-term underlying bond maturity, respectively. Despite their differences in the underlying maturity and market liquidity, all three futures are used as the main hedging instruments for euro interest rate risk.

The tail behavior is worth studying for several reasons. First, it gives new insights into the occurrence of extreme euro market events and its implications for interest rate portfolios. Therefore, it allows financial institutions to assess market risks more accurately and contributes to the overall stability of the euro financial system, which has been immensely challenged since 2008. Second, the occurrence and severity of the financial crisis in 2008 question the underlying assumptions of many risk management techniques for trading activities. Of special relevance is the widely used assumption of a symmetric and normal return distribution, which underestimates the risk of extreme events and does not differentiate for long and short positions. Since activities of single trades have proven to lead to the failures of whole institutions in the past, the correct assessment of tail risk appears to be of great relevance in this context.[2] Third, the identification of cross-sectional differences among the Bund, Bobl and Schatz future allows to the optimize hedging ratios of euro interest rate transactions, which implies not only more effective risk management but also cost reductions. Fourth, since the financial crisis in 2007/08 and the European sovereign debt crisis in 2010/11 the ECB has implemented many conventional and unconventional monetary policy measures, which resulted in a historical low interest rate environment. However, since this is a relatively new market environment, the effects of these policy measures are still not fully understood and require further investigation. As the ECB policy conduct aims, inter alia, at providing stability and reduce market uncertainty, its relation to tail risks in the euro interest rate market is of special interest and requires further research.

My analysis follows a similar approach to Werner and Upper (2002), who study the tail behavior of Bund future returns from 1997 to 2001. The focus is on the tail index α, which measures the speed of the probability decay if one looks deeper into the tails of the return distribution. In general, a lower tail index implies thicker tails and thus,

[2]See the case of Barings Bank in 1995, which collapsed due to the speculation with interest rate and index futures by its trader Nick Leeson in Singapore.

higher tail risks, ceteris paribus. First, I analyze if the daily returns of Bund, Bobl and Schatz futures exhibit fat tails over the period from 1999 to 2016 and if there are significant differences among these three securities with respect to tail risk, as measured by the tail index α. Since the futures' underlying bonds have different maturities and thus, durations, I hypothesize that the long-term Bund future realizes more extreme price fluctuations than the medium-term Bobl and short-term Schatz future. Therefore, the Bund future should also exhibit the highest tail risk, followed by the Bobl future and then the Schatz future.

Second, I investigate if the tail risk of the three future contracts has changed over the period from 1999 to 2016. Due to the massive market interventions by the ECB as response to the financial crisis and sovereign debt crisis, there has been an extreme change in the interest rate level in the euro zone. Therefore, I hypothesize a structural change in the tail indices around 2008, which represent an increase in the probability of extreme returns of the Bund, Bobl and Schatz future. In this case I anticipate that the Bund future is effected to the largest extend due to its highest underlying bond duration. Moreover, I expect that only the right tail of the return distributions indicates the higher probability of extreme returns, since the interest rate level in the euro zone primarily decreased since 2007 and did not rise considerably at any point in time. Due to the negative relation of interest rates and bond prices, extreme positive returns of futures on German government bonds should appear more often than negative ones since 2007.

Third, I analyze if the tail index contains further information beyond the conventional market risk measures volatility and its implication for value-at-risk estimates in case of euro interest rate futures. Since the concept of volatility refers to a symmetric return distribution, the expected increase in the right tails' fatness since 2008 might not be accurately assessed, which can lead to unexpected severe losses of short positions in the future contracts. Furthermore, several value-at-risk methods assume normality in the return distribution, which leads to a significant underestimation of tail events. Thus, I will also investigate the implications of this underestimation in terms of value-at-risk estimates.

Anticipating the results, I provide significant evidence for fat tails of the return distribution of the Bund, Bobl and Schatz future. In contrast to expectations, the results indicate the highest tail risk for the short-term Schatz future and the lowest for the long-term Bund future. Differences in market liquidity might be a factor behind this. Furthermore, I find comprehensive evidence for an increase in right tail risk for all three futures since 2008. As expected, this increase in tail risk is most significant for the long-term Bund future. Surprisingly, also evidence for a decrease in the left tail risks is found, although at lower significance. Furthermore, the analysis presented in this paper indicates that the tail index contains risk information, which is not captured by the concept of volatility. Consequently, the accuracy of value-at-risk estimates for different long and short positions in euro interest rate futures can be improved by taking into account the tail index α explicitly in the estimation process.

The existing literature about the tail risks in the euro interest rate market is rather limited. Although, research about the tail-behavior of the long-term Bund future exists, it does not include the recent developments in the euro market. My analysis contributes to the existing literature in the following ways. First, to the best of my knowledge, I am the first one who investigates explicitly the tail behavior of the Bund, Bobl and Schatz futures since the financial crisis in 2008 and compares it to the situation before. Second, a comparison between these three futures contracts in terms of extreme risks over time has not been conducted before. Therefore, this analysis creates a basis for further research in the area of optimal hedging methods for euro interest rate transactions in the current market environment. Third, based on the contribution of Straetmans and Candelon (2013), I overcome a major shortcoming of the previous work by Werner and Upper (2002) in terms of over-rejection of tail index stability. Fourth, the present unconventional monetary policy of the ECB lacks of a comprehensive understanding of its market-wide effects. This analysis gives new insights in terms of tail risks in the euro interest rate market and the present monetary policy conduct. The results presented in this paper provide evidence for an increase in market risk in the current context of the expansionary monetary policy of the ECB, which represents an argument against its very extension.

In order to analyze these aspects, the rest of this paper is organized as follows. Section 2 gives a review of the existing literature in this field, from which the contribution of this research is further constituted. Based on existing research and their proposed estimation methods for tail risks, the methodology for this study is explained in Section 3. The data series are outlined in Section 4, together with the descriptive statistics. The tail behavior of the futures' return distribution, measured by tail index α, is analyzed and compared in Section 5.1. Structural changes in the tail index over time are identified in Section 5.2. Section 6 answers the question if the tail index contains additional information beyond the concept of volatility and its implications for value-at-risk measures. Section 7 concludes the analysis by summarizing the findings, identifying their limitations and providing the ground for future research in this area.

2 Positioning in the Existing Literature

The distribution of financial returns has been a topic in academic research since decades, see Mandelbrot (1963) for an early analysis. Since this research area is not only theoretical, but also highly relevant for many applications in the financial industry, the literature on return distributions has grown significantly over the years. Consequently, there are many papers covering not only stock returns but also other asset classes like fixed income, foreign exchange or commodities. Since the global financial crisis in 2008, a discussion about the appropriateness of risk management techniques of financial institutions and their underlying assumptions has once again gained momentum. Of special relevance is the market risk management, which also comprises assumptions about the return distribution of financial assets. Here, the focus is on the tails of the distributions and the assessment of extreme events, which threaten the stability of whole financial systems.

Important research has been conducted in the area of extreme returns of bank stocks and the consequences for the stability of the banking system. However, the since 2008 produced literature on extreme events in the euro interest rate market is rather limited, despite its high relevance for the economic system. In case of the Bund, Bobl and Schatz future, this might be due to the lower perceived risk of the underlying bonds and their relatively low volatility. However, as Werner and Upper (2002) argue, this conclusion might be misleading considering the immense leverage in a future position on these bonds. This section highlights the need of more research in this area by giving an overview of the existing literature and analyzing its shortcomings, given the current market environment. Thereby, I position the paper in the current literature and explain its contribution to existing research. In the following I give an overview of the existing literature directly related to the Bund, Bobl and Schatz future. I also introduce conducted research concerning the tail behavior in other asset classes, since research specifically about the Bund, Bobl and Schatz future is limited and the underlying theory and methodology can be generally applied across all asset classes.

2.1 Tail Risk and the Bund, Bobl, and Schatz Future

The analysis presented in this paper is based on the contribution by Werner and Upper (2002). Therefore, I will review their study in more detail compared to other existing literature. The two authors investigate the tail behavior of the returns of the long-term Bund future by the measure of the tail index α during the time period from January 1997 to December 2001. Consequently, it does not comprise the financial crisis in 2007/08, the European sovereign debt crisis in 2010/11 and the resulting monetary policy measures of the ECB, which lead to a historical low interest rate environment in the euro zone. This shortcoming will be addressed in my analysis, which uses a more recent sample period until August 2016. In their analysis Werner and Upper (2002) do not only use daily returns but also intra-day data, specifically, five minute returns and one hour returns. Thereby, differences in the tail behavior for different return frequencies are shown. However, since the future contracts are settled at the end of each trading day, the daily returns are of greater significance for risk management applications than the intra-day returns.

The authors find significant fat tails in the distribution of Bund future returns, which are not constant over time and change due to extreme events like the September 11th attacks. This result only applies to the higher frequencies of five minute and one hour returns and not daily returns. Over the considered time period, higher return frequencies appear to have fatter tails than lower frequencies. Moreover, the authors show that in case of the Bund future the tail behavior of a return distribution is not accurately captured by conventional market risk measures like volatility. Thus, the tail index α contains information, which is beyond standard risk measures.

In contrast, Werner and Upper (2002) do not investigate the tail behavior of the medium-term Bobl future and short-term Schatz future. Therefore, the cross-sectional differences between these securities are unidentified. Although the long-term Bund future has a significantly higher market liquidity, the Bobl and Schatz future are also frequently used for hedging interest rate transactions. This highlights the importance of a better understanding of the tail behavior of all three future contracts. In this paper I overcome this shortcoming and extend the analysis to the Bobl and Schatz future. In their methodology Werner and Upper (2002) use the Hill estimator to calculate the tail index for the Bund future return distribution. Since I apply this methodology in an identical version in my analysis, I explain this method in detail in Section 3 together with the shortcomings of the approach chosen by the two authors.

A related study concerning the distribution of Bund future returns has been conduced by Herlemont (2005). The author analyses the value-at-risk and conditional value-at-risk for a fund trading only the Bund future over a time period from November 1990 to October 2004. Similar to Werner and Upper (2002), Herlemont (2005) finds that the distribution of daily Bund future returns exhibits significant fat tails. He estimates different VaR models based the on the normal distribution, empirical distribution, Corner-Fisher expansion, extreme value theory and GARCH models. He concludes that the conditional VaR should be used as main risk parameter instead of the

VaR, since the latter does not say anything about the actual loss. In this case especially the historical VaR estimates appears to underestimate the actual losses occurring. Using the conditional VaR, the riskiness of the fund will be significantly reduced through decreasing the number of traded contracts by approximately twelve percent. Furthermore, the author proposes to reduce the model risk in the Bund future's VaR estimates by taking the highest estimate across all models. However, the study focuses only on the long-term Bund future and does not analyze the Bobl and Schatz future. Another shortcoming of the study is the outdated sample period, since it does not include the recent financial crisis and European sovereign debt crisis, which questioned the effectiveness of many risk management techniques. In this paper I address these two crises in VaR considerations for the Bund, Bobl and Schatz future.

In a more recent study Bessler and Wolff (2014) investigate how to optimally hedge European government bond portfolios during the sovereign debt crisis. For this a sample of daily observations is used, which ranges from January 2006 to December 2011. In contrast to previous literature, the authors do not only use the long-term Bund future but also the medium-term Bobl and short-term Schatz future as hedging instrument. Additionally, the BTP future on ten year Italian government debt is proposed as hedging instrument against the credit risk underlying European government bond portfolios. The concept of duration and the minimum variance approach are used to determine the optimal hedge ratio. Their study reveals that the Bund, Bobl and Schatz future were effective hedging instruments for EMU bond portfolios before the crisis and for government bonds with low credit risk during the crisis. Nevertheless, during the crisis the hedging effectiveness for EMU bond portfolios is significantly reduced if these three future contracts are used as single hedging instrument. In this case the Italian BTP future is a more effective single hedging instrument, since it also reflected the spike in credit spreads during the years 2010 and 2011, which was not the case for risk-free considered German government bonds. However, the main finding of the authors is that a joint hedge with German and Italian government bond future performs even better during the crisis compared to the single hedges. Bessler and Wolff (2014) conclude that a combination of the Bund, Bobl, Schatz and BTP future is the optimal hedging strategy to reduces the variance and also the tail risk of European government bond portfolios.

To the best of my knowledge, additional public available literature which covers explicitly the tail behavior of the Bund, Bobl and Schatz future does not exist. However, these three future contracts have been studied in a related topic by Fricke and Menkhoff (2011). The authors analyze the price discovery of the interest rate level by using an information share approach for the Bund, Bobl and Schatz future. Since the Bund future has the longest underlying bond maturity and highest market liquidity, one would expect that it dominates the two shorter maturity futures in terms of information shares. However, Fricke and Menkhoff (2011) show that the Bund future is indeed the most important contract in price discovery but does not dominate the two other contracts in each aspect. Instead, the medium-term Bobl and short-term Schatz future contain relevant information shares, which are occasionally even higher than the information share of the Bund future. This highlights the problematic of neglecting the less liquid Bobl and Schatz future. Thus, the findings by Fricke and Menkhoff (2011) provide a convincing argument that a comprehensive and complete market risk assessment should take into account all three future contracts.

The presented literature highlights two important aspects, which are not addressed by the analysis of Werner and Upper (2002). First, for applications in the industry it is essential to have a comprehensive risk assessment, which also includes the medium-term Bobl future and the short-term Schatz future. Second, the recent financial crisis in 2007/08 has challenged many risk management models, which stresses the need of up-to-date research in this area. For a better understanding of the tail risk in the euro interest rate market, these two aspects are considered in the analysis presented in this paper. Additionally, to structurally underpin my topic on a fundamental level, I present in the following existing literature concerning the tail risk in other asset classes. Conveniently, the underlying theory and methodology is applicable to financial returns across all asset classes and thus, also relevant for my analysis.

2.2 Tail Risk and Other Asset Classes

The majority of existing literature focuses on the tail behavior in the asset classes equity and foreign exchange. For the sake of comparability, I focus on the literature which uses a similar methodology to the one applied in my analysis. In the following I present a general overview of these papers, while in Section 3 I present the used methodology in detail. In case of equity, the tail behavior of the returns of bank stocks has become a major interest of academic research. During the financial crisis capital buffers of many financial institutions turned out to be insufficient, which increased the need for significant research about the liability side of banks' balance sheets. A recent study concerning this topic has been conducted by Straetmans and Chaudhry (2015) as an extension to the work by Hartmann et al. (2006). In their study the authors use statistical extreme value analysis to calculate market-based estimates for banks' downside risk and systemic risk. The considered time period starts in April 1992 and ends in June 2011, which includes the financial crisis and the European sovereign debt crisis. The left tail risk is used as indicator for the downside risk of banks' stock returns and a tail-β is introduced as an indicator for systemic risk. Identical to my analysis, the tail risks are measured by the tail index α, which is estimated by the common method of Hill (1975). For a cross-Atlantic comparison of tail risks and systemic stability, banks in the euro zone and in the United States are considered. Their study reveals that the tail risk and systemic risk are higher for banks in the United States than for the euro zone regardless of the year. Furthermore, they find a significant increase in tail risk and systemic risk during the financial crisis. In contrast, the European sovereign debt crisis appears to have only a minor impact on banks' equity capital. Similar to Straetmans and Chaudhry (2015), I investigate differences in market risk of euro interest rate futures during the financial crisis and sovereign debt crisis, since the crises underlying market mechanisms appear deviant.

Another study of the tail behavior in the asset class equity is provided by Lux (2001), who uses extreme value theory and intra-day data for the German stock index Dax 30 from 1988 to 1995. Similar to the other studies, the tail index estimation method by Hill (1975) is applied with several approaches to determine the optimal tail fraction in the return distribution. The analysis confirms previous studies and estimates a range for the tail index for the Dax index between three to four. Previous work by Müller et al. (1998), who use an alternative approach to define the extreme part in the tail of a return distribution, yields similar results. This confirms their findings robustness to the optimal choice of the tail fraction. Based on this approach, I apply in my analysis more than one method to determine the optimal sample fraction of extremes, which allows me confirm the robustness of my findings. Additionally, the study by Lux (2001) finds no evidence for an unstable tail behavior of Dax returns over the considered time period. These results are consistent even at lower data frequencies. In contrast, Jondeau and Rockinger (2003) look at several stock indices around the world and investigate differences in tail indices. Contrary to previous research, their results suggest no significant differences in the tail indices for the left and right tail of the return distribution. Moreover, their tail index estimates appear to be similar for the considered stock indices of different countries.

In case of the tail behavior of foreign exchange rate returns early research has been conducted by Koedijk, Schafgans and de Vries (1990), who also base their analysis on extreme value theory. In this study the level of tail fatness and parameter stability for bilateral EMS foreign exchange rates is investigated over a period from to 1971 to 1987.

To determine the tail fatness the authors estimate the tail index α based on the method of Hill (1975). To test for the parameter stability, the tail index α is estimated over a pre-EMS period and EMS period. Identical to previous work, their results indicate significant fat tails for EMU exchange rate returns, which appear to be stable over time. In other words, this provides evidence that EMS did not lead to reduction of extreme exchange range volatility. Moreover, the study aims at identifying the most appropriate distribution assumption for these returns series. With estimated tail indices around the value two, their study contradicts previous work by Boothe and Glassman (1987), who estimate tail indices significantly above three and even find evidence related to a normal distribution. In later work Koedijk, Stork and de Vries (1992) study this subject in more detail. In my analysis I will follow a similar approach to Koedijk et al. (1990) and estimate the tail index α over two sub-samples, one before the financial crisis 2008 and one after. However, in contrast to the authors, I will also apply a more formal test for structural changes in the tail index by Quintos et al. (2001) to confirm results in a more rigorous manner.

A multi-asset analysis of the tail risks over time is given by Straetmans and Candelon (2013). Their work constitutes an extension and refinement of the study by Quintos et al. (2001) and their previous paper Candelon and Straetmans (2006). In their empirical application the authors test for structural changes in the tail index for stock markets, bond market, currencies and commodities with a differentiation between developed and emerging markets. They show that stock market tails of emerging markets are not more prone to change over time than for developed countries. In contrast, this does not hold for currencies of emerging markets. The work by Straetmans and Candelon (2013) differs from the approach of Quintos et al. (2001) in two important aspects, which increase the accuracy and robustness of the findings. First, the optimal number of extremes in the tail is estimated through the minimum of the asymptotic mean squared error of the tail index, instead of using a fixed fraction of the extremes from the total sample. Second, the authors propose to use bootstrap-based critical values in tests for structural changes in the tail index. These take into account the bias in the estimates and solve the problem of over-rejecting the hypothesis of a stable tail index.

Galbraith and Zernov (2004), who study the tail index of US equities, also apply the structural change test by Quintos et al. (2001). They find strong evidence for a decrease in the tail index during the introduction of large scale program trading and circuit breakers. However, their conclusion is compromised by the tendency of over-rejecting the hypothesis of a constant tail index. This is caused by too low asymptotic critical values, which do not take into account the bias in the tail index estimate. In my analysis I also apply the structural change test by Quintos et al. (2001). However, I implement the augmented version of this test based on the proposals by Straetmans and Candelon (2013). In the following section the structural change test by Quintos et al. (2001) as well as the popular tail index estimation method by Hill (1975) are explained. Furthermore, I introduce several methods to estimate the optimal number of extremes in the tail, which allow me to confirm the robustness of my results.

3 Methodology

In order to test if there are difference in the tail behavior over time and between the Bund, Bobl and Schatz future, I will use the following approach. First, I present arguments for using a semi-parametric method. Second, I explain how I estimate the fatness of tails by the widely used concept of Hill (1975). Third, since the approach by Hill (1975) crucially depends on certain parameters, I will introduce several ways how to estimate these accurately. Fourth, in order to investigate if the tail risk of the euro interest rate futures has increased since 2008, I introduce the recursive test of Quintos et al. (2001), which allows to identify structural changes in the tail index. Since my analysis is based on the work by Werner and Upper (2002), I will use a similar methodology, which allows me to compare my results to the existing work in this field.

3.1 Fat Tails and the Hill Estimator

Since the work by Mandelbrot (1963) it has been continuously proven that returns of financial securities rarely follow a normal distribution. Instead, a typical characteristic of financial time series is the exhibition of fat tails. This implies that there is a larger probability of extreme observations in comparison to the normal distribution. Nevertheless, many financial models in academics and in the industry rely on the assumption of normality. Based on the Central Limit Theorem, the normal distribution is an accurate approximation of the real distribution in terms of sums and averages. However, when assessing the stability of euro interest rate future returns, the normality assumption leads to a serious underestimation of extreme events and therefore, is inappropriate in this area. Although there are several other parametric models which do incorporate fat tails, there is still a significant drawback. By making assumptions about the complete distribution range, one imposes significant model risk to the analysis, since the real return distribution of financial assets is unknown. As an alternative to the parametric models, one can use the empirical probability density function of financial returns. However, this requires a sufficiently large historical sample size, which is not always available. Consequently, due to the limit of historical samples, the analysis of extreme observations far in the tails might lack of accuracy and appears inappropriate for stability tests.

However, there exists an elegant way to combine the parametric and empirical approaches and to overcome their individual limitations. This is the semi-parametric approach based on extreme value theory.[3] Instead of making assumptions about the whole distribution range, the semi-parametric approach only assumes a certain distribution for the tails and uses the empirical probability density function for the remaining part. Huisman et al. (2001) argue that all fat tail distributions can be approximated by the Pareto distribution deep in the tails. Thus, I assume that the returns in the right tail of the three future contracts follow a Pareto-type distribution function:

$$1 - F(x) = x^{-\alpha}\mathcal{L}(x) \qquad \alpha > 0 \tag{3.1}$$

In this case $\mathcal{L}(x)$ is a slowly varying function with $\underset{x\to\infty}{lim}\frac{\mathcal{L}(\lambda x)}{\mathcal{L}(x)} = 1$ for all $\lambda > 0$. The parameter α refers to the tail index and indicates the speed at which the mass in the tail decreases as one looks deeper into the tail. It is important to note that, keeping x constant, a lower tail index implies more mass in the tail, i.e. the probability of extreme returns is higher. In contrast, a higher tail index implies less probability mass in the tail and thus, lower tail risk, ceteris paribus. Mathematically the probability of extreme positive returns larger than x is given by $P(X > x) = 1 - F(x) = \bar{F}(x)$. Conveniently, the tail index α also indicates the existing number of moments. Therefore, a tail index smaller than four implies an infinitely high kurtosis. In contrast, under a normal distribution the tail index should in theory approach infinity.

Instead of aiming at a specification of the real parametric tail distribution of Bund, Bobl and Schatz future returns, I assume a Pareto-type distribution in the tails and estimate directly the tail index for each future contract. In general, a great advantage of this semi-parametric approach is that it allows to compare estimates based on different tail sizes. Since the aim of this analysis is to investigate the tail behavior, the negligence of information about the center of the distribution does not represent a disadvantage. There are several ways to estimate the tail index α of equation (3.1).[4] However, I will

[3] A general analysis about extreme value theory and risk management methods is given Embrechts et al. (1999). For a more detailed discussion of the advantages and disadvantages of using extreme value theory for risk management purposes, see Diebold et al. (2000).

[4] Dacorogna et al. (1996) give a general overview of tail index estimation methods.

focus on the method introduced by Hill (1975), the so-called Hill estimator. Through its straightforward implementation, this method is widely used and serves as benchmark in many applications (Matthys and Beirlant, 2000). The Hill estimator is given by the following function:

$$\hat{\alpha}_{Hill} = \frac{1}{\hat{\gamma}} = \left(\frac{1}{k} \sum_{i=1}^{k} ln \frac{x_{n-i+1}}{x_{n-k+1}} \right)^{-1} \tag{3.2}$$

with $x_1 \leq x_2 \leq ... \leq x_n$ equal to the ascending order statistics of the empirical return observations of the Bund, Bobl and Schatz future. In this form the Hill estimator is a maximum likelihood estimator for a Pareto-type distribution of the right tail. The parameter k in equation (3.2) refers to the number of extreme return observations considered in the tail. Therefore, assuming Pareto-type behavior in the tail of the return distribution, one only produces estimations about the area in the tail, which does not exceed k.

The described methodology of this semi-parametric approach refers to positive returns. However, I am interested in both sides of the return distribution. By estimating the tail index for each side of the distribution separately, I can differentiate the risk of extreme returns for different long and short positions in the future contracts. The expectation of different risks for long and short positions is supported by the fact that call and put options with same characteristics appear to have different implied volatilities.[5] A convenient way to determine the tail index for the left side of the distribution with the Hill estimator (3.2) under the Pareto-type distribution assumption is simply to reverse the signs of the historical return series. The right tail describes then the actual negative returns and allows to reduce the computational burden for comparing the risk of long and short positions.

Despite its easy implementation, the Hill estimator has one significant drawback. Its estimates depend crucially on the number of extremes in the sample k. When choosing the optimal k, one is confronted with a trade-off. Increasing the number of extremes leads to a reduction in the estimator's variance. However, at the same time, including more observations which are less "extreme", also leads to an increased bias in the estimator. Consequently, the right choice of k is an important part of this analysis and will be discussed in the following.

[5] For a detailed analysis of fat tails and volatilizes smiles in options, see Duan (1999).

3.2 Determining the Optimal Number of Extremes in the Tail

The literature about determining optimal number of extremes k in the sample has grown significantly over the past decades.[6] However, there is still no clear solution to this question. In order to verify the robustness of my tail index estimations, I will apply several approaches. First, I will apply the widely used method of Beirlant et al. (1999), which uses an exponential regression based estimator to quantify the variance-bias trade-off as a sample equivalent. As Straetmans and Candelon (2013) argue, the general principle underlying this method constitutes common practice in extreme value theory and thus, allows me to compare my results to previous work outlined in Section 2. Second, I will visualize this variance-bias trade-off through the conventional Hill plot, which allows me to confirm my previous results.

3.2.1 Estimates based on the Exponential Regression Model

The idea of the algorithm by Beirlant et al. (1999) is to quantify the variance-bias trade-off through the asymptotic mean squared error of the Hill estimator and choosing its minimum.[7] For this Beirlant et al. (1999) derive the following exponential regression model for the log spacing of the order statistics:

$$j(log\, x_{n-j+1,n} - log\, x_{n-j,n}) \sim \left(\gamma + b_{n,k}\left(\frac{j}{k+1}\right)^{-\rho}\right) \cdot f_j \qquad 1 \leq j \leq k \qquad (3.3)$$

where $b_{n,k}$ is equal to $b\left(\frac{n+1}{k+1}\right)$ with $1 \leq k \leq n-1$. Moreover, $(f_1, f_2, ...f_k)$ is a vector of independent standard exponential random variables. In this case the inverse of the Hill estimator (3.2) represents a maximum likelihood estimator for γ in the following simplified model:

$$j(log\, x_{n-j+1,n} - log\, x_{n-j,n}) \sim \gamma \cdot f_j \qquad 1 \leq j \leq k \qquad (3.4)$$

It can be shown that the Hill estimator is asymptotically normal when k grows at a sufficiently lower rate than n, so that the ratio $\frac{k}{n}$ converges to zero in infinity. Beirlant et al. (1999) argue that the bias in model (3.4) arises through excluding the regression

[6] For an overview of the main types of adaptive threshold selection methods, see Matthys and Beirlant (2000).

[7] Alternatively, Peng (1998) introduces a criterion which is based on the minimum of the MSE of the quantile estimator.

terms of the full model (3.3). The authors show that this bias can be approximated by:

$$Bias(\alpha_{k,n}^{Hill}) \sim \frac{b_{n,k}}{1-\rho}$$

Additionally, the variance of the Hill estimator is approximated by:

$$Var(\alpha_{k,n}^{Hill}) \sim \frac{\alpha^2}{k} \tag{3.5}$$

Thus, the variance-bias trade-off can be calculated as the asymptotic mean squared error of the Hill estimator:

$$AMSE(\hat{\alpha}_{k,n}^{Hill}) = AVar(\hat{\alpha}_{k,n}^{Hill}) + ABias^2(\hat{\alpha}_{k,n}^{Hill}) = \frac{\hat{\gamma_k}^2}{k} + \left(\frac{\hat{b}_{n,k}}{1-\hat{\rho}_k}\right)^2 \tag{3.6}$$

From this the estimator for the optimal number of extremes in the tail $\hat{k}$ can be identified as the point where the asymptotic mean squared error of the Hill estimator reaches its minimum:

$$\hat{k}_{n,2}^{opt} = \underset{3\leq k\leq n}{argmin} \left(\frac{\hat{\gamma}^2}{k} + \left(\frac{\hat{b}_{n,k}}{1-\hat{\rho}}\right)^2\right)$$

In order to determine the optimal fraction of extremes in the distributions of the Bund, Bobl and Schatz future returns, I estimate the exponential regression model (3.3) and determine $\hat{\gamma}_k$ and $\hat{b}_{n,k}$ with $3 \leq k \leq 500$ for each tail side of three future contracts.[8] Identical to Matthys and Beirlant (2000), I do not estimate ρ, but fix the value at -1 throughout my analysis. The authors argue that for many distribution fixing this parameter performs better, with respect to the MSE, than estimating it, even if specified inaccurately. This also corresponds to the findings of Drees and Kaufmann (1998). With the resulting estimates $\hat{\gamma}_k$ and $\hat{b}_{n,k}$ I calculate the asymptotic mean squared error of the Hill estimator with equation (3.6) for each k. Based on the minimum of the $AMSE(\hat{\alpha}_{k,n}^{Hill})$, I get an estimate for the optimal sample fraction of extremes k for each tail side of the distributions. With these $\hat{k}_{n,2}^{opt}$ I calculate the tail index for the daily return distributions of the Bund, Bobl and Schatz future according to the Hill estimator (3.2).

[8] I reduce the computational burden and do not estimate model (3.3) for $k \geq 500$. Given the total sample sizes in my analysis, this does not affect to the quality of the results.

3.2.2 The Hill Plot

Additional to the quantitative approach of Beirlant et al. (1999), I produce Hill plots for each future's tail side. This allows me to visualize the variance-bias trade-off and to confirm my results from the previous method. The Hill plot is a simple and effective method to determine the optimal number of extremes through the so-called "Eye-Balling technique". The Hill estimates are plotted for all possible values of k in an ascending order. The optimal number of extremes is then estimated visually in the area where the variance of Hill estimator is sufficiently low i.e. where $\hat{\alpha}$ appears stable to the choice of k and the bias is not too large.

The major disadvantage of this heuristic method is that the results are subjective and not based on quantitative analysis. Consequently, an exact determination of the optimal number of extremes is difficult and the results might be inaccurate. Since the Hill estimator is highly sensitive to the choice of k, there is significant potential for estimation errors. However, I will use the estimates of Hill plot only as a supportive argument to verify the results from the Beirlant et al. (1999) method.

In contrast, Werner and Upper (2002) do not use the Hill plot to verify their results but the regression approach of Huisman et al. (2001). Their method uses a weighted average of a set of Hill estimates, each based on a different number of extremes in the tail. The weights are determined through OLS estimation. In this approach the number of extremes is a constant fraction of the sample size $\kappa = \frac{k}{n}$. However, Straetmans and Candelon (2013) and Dumouchel (1983) argue that under this assumption the Hill estimator lacks of asymptotic normality, since the number of extremes does not increase at a sufficiently low rate as the overall sample size grows. Therefore, I will not use the method of Huisman et al. (2001) to verity the results of the other methods. Nevertheless, for sake of completeness and comparability to Werner and Upper (2002), I report tail index estimates based on the Huisman et al. (2001) method in the appendix.[9]

3.3 Identifying Structural Changes in the Tail Behavior

In order to investigate if the tail behavior of the Bund, Bobl and Schatz futures changed during the financial crisis and sovereign debt crisis, I apply the method by Quintos, Fan and Phillips (2001). In their analysis the authors use three different tests to identify structural changes in the tail behavior, which are the recursive, sequential and rolling test. Each of them is based on a different division method of sub-samples for the Hill

[9] A detailed description of the used methodology of this approach is available upon request from the author.

estimator. However, in simulations Quintos et al. (2001) show that the recursive test dominates in identifying change dates and in finite sample power. Consequently, I will only apply the recursive test, which's statistic is calculated according to the following:

$$Y_n^2(r) = \left(\frac{t \times k_t}{n}\right)\left(\frac{\hat{\alpha}_t}{\hat{\alpha}_n} - 1\right)^2 \tag{3.7}$$

with $r = \frac{t}{n}$ as in increasing fraction of the full sample and $\hat{\alpha}_n$ equal to the Hill estimate for full sample n. The sample size r is increased successively by one daily observation. The recursive version of the Hill estimator (3.2) is defined as:

$$\hat{\alpha}_t = \frac{1}{\hat{\gamma}_t} = \left(\frac{1}{k_t}\sum_{i=0}^{k_t-1} ln\left(\frac{x_{t-i,t}}{x_{t-k,t}}\right)\right)^{-1} \tag{3.8}$$

In order to estimate k_t I follow the approach by Straetmans and Candelon (2013). In this case, $\hat{k}_t = \hat{c}t^{\frac{2}{3}}$ with $\hat{c} = \frac{\hat{k}_n}{n^{\frac{2}{3}}}$ as the full sample scaling constant. This ensures that $\hat{k}$ grows at a sufficiently low rate as n increases. In my analysis I calculate $\hat{c}$ for each tail side of the Bund, Bobl and Schatz future. I determine the optimal number of extremes in the full samples $\hat{k}_n$ according to the algorithm by Beirlant et al. (1999).

The hypothesis of a constant tail behavior of euro interest rate futures over time can be formally expressed as:

$$H_0 : \alpha_{[t]} = \alpha \qquad H_A : \alpha_{[t]} \neq \alpha$$

The null hypothesis of constancy is most likely to be rejected at the time where the estimated recursive test statistic reaches its supremum:

$$Q = \sup_{r \in R_\tau} \; Y_n^2(t)$$

under the condition that Q is greater than the critical values for the 95% and 99% confidence level. Werner and Upper (2002) use the asymptotic critical values by Quintos et al. (2001). However, Straetmans and Candelon (2013) provide convincing evidence that the critical values depend on the tail parameters of the return distribution. The asymptotic critical values by Quintos et al. (2001) do not reflect the bias in the Hill estimator and are too low. Consequently, the null hypothesis of tail index constancy will be widely over-rejected. In other words, Straetmans and Candelon (2013) argue that each return series requires a unique set of critical values, which take into account the bias in the Hill estimator.

As proposed by the two authors, I use bootstrapped-based critical values at the 95% and 99% confidence level. To determine these, I estimate a GARCH(1,1) model for the Bund, Bobl and Schatz return series, respectively. From this I extract the estimated daily volatilities. To create 10,000 return sample replications for each of the three futures, I multiply the extracted daily volatiles with by random variables which follow $\mathcal{N}(0,1)$. The results are 10,000 replicated samples of 4489 daily return observations for each of the three future contracts. I use these sample replications to calculate the simulated recursive test statistics by the described methodology of Quintos et al. (2001). From the resulting simulated distribution of bootstrap-based recursive test statistics, I use the 1% and 5% quantiles as my critical values, respectively. Since I use different values for $\hat{k}_n$ for each tail side in the estimation of the recursive Hill estimator (3.8), the bias will be different for each tail side. Thus, I determine separate critical values for the left and right tails of the three futures' return distributions. This results in twelve bootstrap-based critical values for the 95% and 99% confidence level.

Since there were significant changes in monetary policy during the financial crisis, I expect the supremum of the recursive test statistics Q to occur during the years 2008 or 2009. Furthermore, since I cannot exclude the possibility of a decreases in the tail risks for Bund, Bobl and Schatz future, I perform also a backward version of the recursive test, as implemented by Straetmans and Candelon (2006, 2013). For this I simply revert the sample, with the latest observation being the first, and re-estimate the recursive test statistics (3.7) and Hill estimates (3.8). While the regular "forward recursive test" signals decreases in the tail index, the "backward" version indicates increases in the tail index. In the following section I describe in detail the data series I use for these test.

4 Data Series

Euro interest rate futures have been listed since October 1998. They are traded on the European Exchange (Eurex), which is specialized in derivatives like options and futures. The trading hours are from 8:00 to 22:00 (CET) on regular working days. Officially the Bund, Bobl and Schatz future belong to the wider category of fixed income futures. However, since the underlying German government securities are considered as risk free, these futures only reflect the interest rate level in the euro zone and no credit conditions. This is in contrast to other fixed income futures like the long-term euro BTP (Buoni del Tesoro Poliennali) or euro BONO (Obligaciones del Estado) future, which underlying securities are Italian and Spanish government bonds, respectively. Since the aim of this analysis is, inter alia, to investigate cross-sectional differences among the Euro Bund, Bobl and Schatz future, it is important to differentiate precisely the underlying bonds of these futures, as done in the following.

4.1 The Bund, Bobl and Schatz Future

In general, within future contracts there has to be a differentiation between contract size and price. The contract size refers to the deliverable quantity of the underlying asset. In case of one Bund, Bobl and Schatz future this is equal to EUR 100,000 of German government bonds with the respective maturity.[10] The prices of these futures are quoted as percent of the par value. For instance, a quotation of 150.00 for one future refers to a price of EUR 150,000 for EUR 100,000 face value plus coupons of a German government bond with the respective maturity. Consequently, the tick value of this future is EUR 10.

Specifically, the long-term Euro Bund (Bundesanleihe) future has an underlying German government bond with a remaining maturity of ten years and a coupon rate of 6%. However, since on delivery day there is not always a bond with these characteristics available in the market, the underlying bond is actually fictitious. Alternatively, the short position in the future contract can deliver a German government bond, which is equivalent to the one specified in the future. In case of the Bund future, deliverable eligible bonds have a remaining maturity between 8.5 and 10.5 years, an original maturity of no longer than 11 years and minimum issue amount of EUR 5 billion. Based on the so-called conversion factor, the delivered amount of the eligible bond is adjusted in such a way that it matches value of the fictitious bond underlying the future. Since there are usually several eligible bonds, the short position will always choose the one which is the cheapest to deliver. Furthermore, it is important to know that the long-term Bund future is widely used as an indicator about the future interest rates levels in the euro zone. Due to the negative relation between bond prices and interest rates, a price increase in the Bund future indicates market expectations about falling interest rates, while a price decrease signals a rise in interest rates.

In case of the medium-term Euro Bobl (Bundesobligation) future the underlying German government bond has maturity of five years and a coupon rate of 6%. Again, on delivery date this underlying bond is rarely available in the market and therefore,

[10] Before the introduction of the euro in January 1999, the three futures had a contract size of DM 250,000 of the respective German government bond with a coupon of 6%.

actually fictitious. In this case alternative eligible German government bonds for delivery must have a remaining maturity of 4.5 to 5.5 years. The short-term Euro Schatz (Bundesschatzbrief) future has an underlying German government bond maturity of two years with a coupon rate of 6%. Here, for delivery eligible German government bonds must have a remaining maturity of 1.75 to 2.25 years.

In general, the three interest rate futures have expirations up to nine months and expire in March, June, September and December of each year. Depending on the expiration date there are price differences in the futures. For my analysis I will only look at the most actively traded contract, which is generally the one with the nearest expiration date. This future is rolled over on the last Thursday of the trading period. The last trading day is two exchange days prior to the delivery day of the relevant contract. Conveniently, Bloomberg offers adjusted time series price data, which links all most active contracts over a desired time period. Additionally, it should be noted that there is also Euro Buxl (Bundesanleihen extra large) future, where the underlying fictitious German government bond has a maturity of 30 years and a coupon rate of 6%. Nevertheless, since this contract is significantly less traded in the market and hardly used for hedging purposes, I will not include it in the analysis and only focus on the three main futures Bund, Bobl and Schatz.

4.2 Retrieving and Transforming the Data

In order to investigate the tail behavior of euro interest rate futures, I retrieve daily closing price data for the most active Euro Bund, Bobl and Schatz futures from Bloomberg, which is already adjusted for rollovers. The time period for my analysis starts January 4th 1999 and ends on August 22nd 2016. This implies a total sample size of 4490 daily price observation for each future. From these I calculate the daily logarithmic returns by $r_t = ln\left(\frac{Price_t}{Price_{t-1}}\right)$. Using the natural logarithm of returns has the advantage of time-additivity. Moreover, for all three future contracts the returns satisfy $r_t \ll 1$, implying approximate equality of log and raw returns. The presented calculations and figures in this paper have been produced in MATLAB R2015b and Microsoft Excel 365.

The left side of Table 1 shows the descriptive statistics of the daily returns of the Bund, Bobl and Schatz future over the whole sample period from January 1999 to August 2016. All three futures have a slightly positive daily mean return, with the Bund future having the largest and the Schatz future the smallest close to zero. Concerning the minimum and maximum daily return, the Bund future dominates again and is followed by the Bobl future. Also, the daily volatility is the largest in case of the Bund

	Bund	Bobl	Schatz	Bund	Bobl	Schatz	Bund	Bobl	Schatz
	01/1999 - 08/2016			01/1999 - 07/2008			08/2008 - 08/2016		
Mean Return	0.008	0.005	0.001	-0.001	-0.001	-0.001	0.019	0.011	0.004
Max Return	1.959	1.238	0.577	1.271	0.956	0.577	1.959	1.238	0.527
Min Return	-2.267	-1.722	-0.737	-1.490	-1.197	-0.737	-2.267	-1.722	-0.612
Volatility*	0.359	0.229	0.088	0.320	0.216	0.095	0.401	0.243	0.078
Kurtosis	5.50	9.42	10.37	4.29	5.61	8.45	5.73	12.14	14.05
Skewness	-0.40	-0.94	-0.69	-0.41	-0.64	-0.74	-0.42	-1.20	-0.51
Mean Volume**	798	453	396	877	470	417	705	433	372

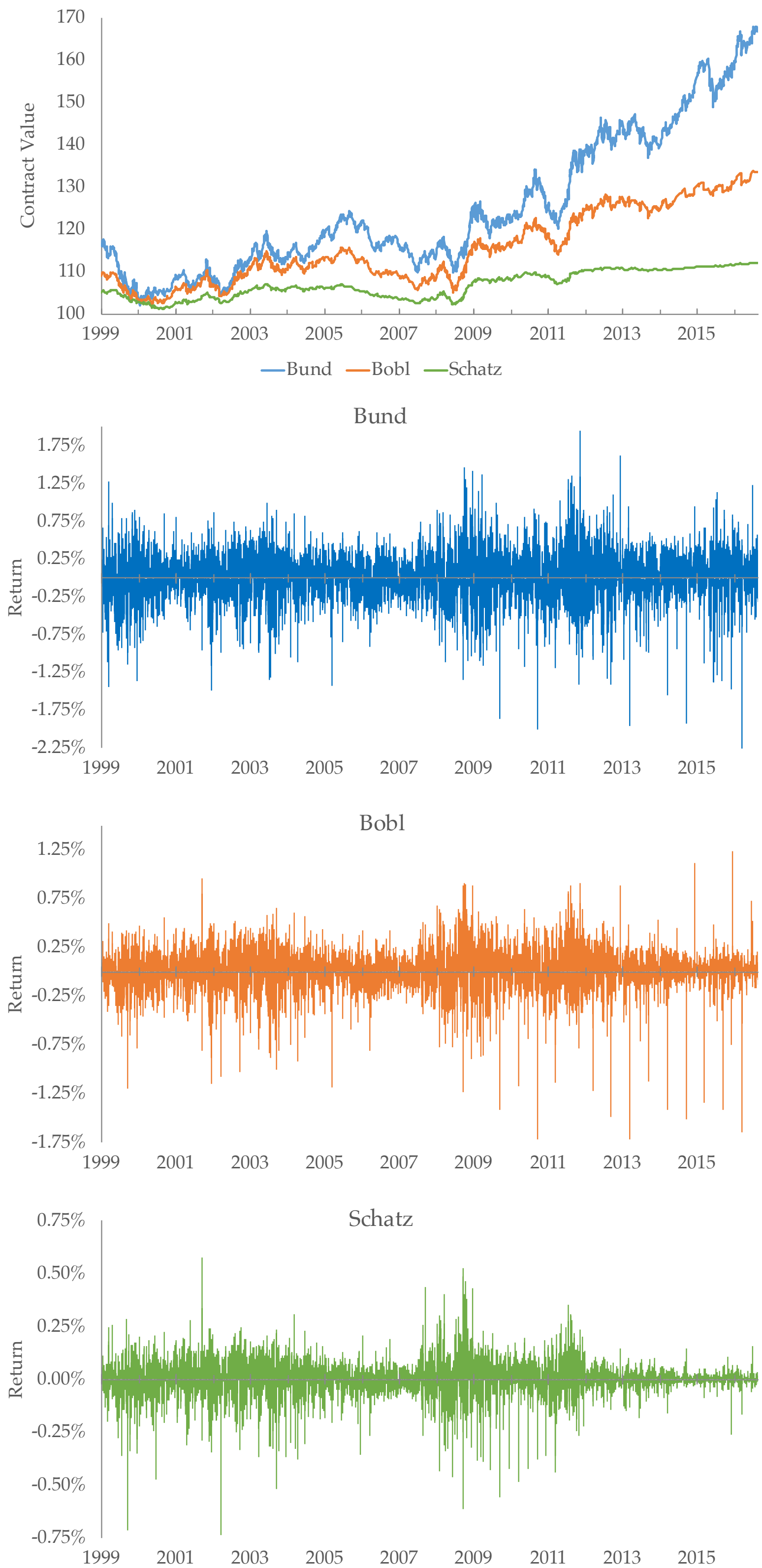
Notes: The presented figures are based on daily frequency. The returns are calculated as log returns and presented as percentage.
*Refers to the daily standard deviation of returns as percentage.
**The mean volume refers to the average daily trading volume in thousands.

Table 1: Descriptive Statistics of Daily Future Returns

future and the smallest for the Schatz future. This indicates that, in terms of market risk, a position in the Bund future has historically the highest risk. However, from the volatility the risk of a short and long position cannot be differentiated and therefore, can lead to wrong conclusions. Moreover, the kurtosis value of all three futures is clearly above the normal distribution value of three, which implies fat tails. Surprisingly, the short-term Schatz future has the highest kurtosis value and the long-term Bund future the lowest value. This can be seen as a first indication for cross-sectional differences in the tails of the return distribution between the Bund, Bobl and Schatz future.

Nevertheless, it should be noted that there are several drawbacks in using the kurtosis as a measure of the tail heaviness. As Brys, Hubert and Struyf (2006) argue, there is no general agreement on what kurtosis actually estimates, since it is also used to measure the peakedness of a distribution. Analogously to the volatility, the kurtosis value also does not allow to differentiate the fatness of the left tail and right tail of the return distributions. Due to its restriction to symmetric distributions, the fourth moment is an inappropriate market risk measure of extreme events for different long and short positions. This stresses the importance of a detailed analysis, which considers the tails of the return distributions separately, as performed in this paper.

In terms of symmetry, all three return series are skewed to the left. Especially, the medium-term Bobl future has a negative skewness. However, from the skewness it cannot be clarified if the left tail is fatter or just longer than the right tail. Together with the kurtosis, the presented values are already a first sign of differences in the tails among the three future contracts, which will be investigated further in this paper. Furthermore, it can also be seen from Table 1 that over an average day the Bund future



Notes: The contract value of the three futures in the upper part of the figure is shown in thousands of euros.

Figure 1: Price Chart and Daily Future Returns over Time

is significantly more traded than the Bobl and Schatz future and therefore, has the highest market liquidity.

In order to analyze the price development over time, Figure 1 shows the price chart and the daily returns of the three futures over the whole sample period. Usually, due to the longest underlying bond maturity, the Bund future has the highest price. Since the middle of 2008, the contract prices of all three futures rose, especially of the long-term Bund future.[11] Moreover, from the three lower graphs in Figure 1 it appears that the number of extreme returns has increased since 2008 for all three future contracts. It seems that this increase has been persistent until today, except for the Schatz future. Additionally, there has been relatively high volatility and also clustering for all three futures around the financial crisis in 2008 and the European sovereign debt crisis in 2011. In case of the Bund future the volatility is also clustered in the beginning of 2015, when the ECB implemented its quantitative easing program.

Since the purpose of this analysis is, inter alia, to study the time variation the tail behavior of euro interest rate futures, I follow the approach by Koedijk, Schafgans and de Vries (1990) and create two sub-samples. The first sub-sample only contains the returns prior to the collapse of Lehman Brothers and the resulting interventions by the ECB. Since the ECB already indicated in late August 2008 to lower the key interest rates, I define the first sub-sample period from January 1991 to July 2008. The second sub-sample period is from August 2008 to August 2016 and contains all returns following the financial crisis, the European sovereign debt crisis and all related monetary policy measures. Table 2 shows the resulting sub-sample sizes for each future and the respective return distribution side.[12] Each tail side of the respective future contract and sub-sample has roughly 1000 observations. The first sub-sample is slightly larger than the second sample, due to an additional year of consideration. However, using the methodology described in Section 3, the outcome of the empirical study should not be effected by this minor difference, as long as the number of extremes k in the tails grows at an appropriate low rate with the sample size n (Diebold et al., 2000).

The middle and right part of Table 1 show the descriptive statistics for the first sub-sample and the second sub-sample, respectively. There are significant differences in all statistics over the two sub-periods. Before August 2008 all three futures have a negative mean return of 0.001%, while after this date the daily mean returns are significantly positive, especially for the Bund future with 0.019%. Moreover, regarding

[11]On June 14th 2016 the price of the Euro Bund future reached for the first time more than 165.00, implying a negative yield on a ten-year German government bond.

[12]Through the exclusion of zero percent returns, approximately 3% of the observations are lost.

	Bund Sample Size		Bobl Sample Size		Schatz Sample Size	
Sample Period	Left Side	Right Side	Left Side	Right Side	Left Side	Right Side
01/1999 - 07/2008	1156	1252	1153	1242	1132	1211
08/2008 - 08/2016	929	1098	898	1104	887	1039
Total	2085	2350	2051	2346	2019	2250

Notes: The sample sizes refer to daily observations.

Table 2: Sub-Sample Periods and Sizes

extreme returns, the maximum and minimum returns of the full sample occurred after July 2008, except for the Schatz future. Similarly, in case of the Bund and Bobl future, the volatility indicates a larger range of price fluctuations in the second sample. In contrast, the short-term Schatz future has a higher daily volatility before August 2008.

The kurtosis value for all futures increases in the second sample, which indicates even more probability mass in the tails, compared to the normal distribution. As before, the Schatz future has the highest kurtosis value, followed by the Bobl future. Similarly, the return distributions of the Bund and Bobl future are more negatively skewed in the second period, although only marginally in case of the long-term Bund future. However, the short-term Schatz future is less negatively skewed in the second period. Together with the higher kurtosis values, this represents a first sign of variation in the tail behavior over time for all three future contracts, which is investigated further. Additionally, the average daily trading volume is lower in the second sample period for all contracts, but especially for the Bund future. This might be due to tighter market regulations introduced since the financial crisis.

The presented descriptive statistics for the two sample periods show that the moments of the returns change over time. Consequently, the distribution of the returns does not appear to be constant. With respect to the methodology explained in Section 3, through the sample division into two sub-periods I can investigate in a more rigorous manner if the tail behavior of the Bund, Bobl and Schatz future has changed since the financial crisis and all related conventional and unconventional monetary policy measures by the ECB.

5 Empirical Results for the Tail Behavior

I want to investigate whether the tail risk of euro interest rate futures has increased since the financial crisis in 2008 and the resulting loose monetary policy of the ECB. Moreover, I would like to identify if there are differences in the tail risks between the Bund, Bobl and Schatz future. First, I will present the Hill estimates for each future contract over two different sub-sample periods. Second, I will show the results of the recursive test by Quintos et al. (2001) to identify potential change dates and thereby, verify the differences in the Hill estimates from the previous part.

5.1 Comparing Tail Index Estimates

Table 3 shows the Hill estimates based on equation (3.2) for the tail indices of the Bund, Bobl and Schatz future returns. Here, I differentiate between the left tail and right tail to determine the risk of long and short position in the futures separately. Moreover, Table 3 presents Hill estimates over three different sample periods. The full sample uses the data range from January 1999 to August 2016. The first sub-sample is based on returns from January 1999 until July 2008. The second sub-sample only contains information from August 2008 to August 2016. The optimal sample fractions of extremes are based on the algorithm of Beirlant et al. (1999) and presented for each sub-sample in round brackets behind the estimates. Moreover, Table 3 shows the 95% confidence intervals for the Hill estimates below each estimate in square brackets. Given the approximation for the variance of the Hill estimator in equation (3.5) on page 16, I calculate the confidence intervals by:

$$CI(\hat{\alpha}) = \hat{\alpha} \pm z\frac{\hat{\alpha}}{\sqrt{\hat{k}}} \tag{5.1}$$

In general, a lower tail index implies more mass in the tails and thus, a higher probability of extreme returns, ceteris paribus. Additionally, the tail index is related to the number of existing moments. There are several aspects in Table 3, which are important to note. Considering first the full sample from 1999 to 2016. All tail indices indicate that the returns of the three future contracts have fat tails. In theory, under a normal distribution the tail indices should approach infinity. However, based on the upper bound of the confidence intervals the majority of the estimated tail indices are smaller than six. Especially in case of the left tail of the short-term Schatz future, the estimates indicate an infinitely high kurtosis, since the upper bound of the confidence interval is smaller than four. Furthermore, in all three futures the right tail index

	Bund Tail Index		Bobl Tail Index		Schatz Tail Index	
Sample Period	Left Tail	Right Tail	Left Tail	Right Tail	Left Tail	Right Tail
01/1999 - 08/2016	4.12 (23)	4.21 (19)	3.87 (18)	3.89 (16)	3.08 (47)	3.18 (43)
	[2.44, 5.80]	[2.32, 6.10]	[2.08, 5.66]	[1.98, 5.80]	[2.20, 3.96]	[2.23, 4.13]
01/1999 - 07/2008	4.45 (12)	6.01 (30)	3.81 (21)	4.38 (92)	2.69 (44)	3.31 (123)
	[1.93, 6.97]	[3.86, 8.17]	[2.18, 5.44]	[3.49, 5.28]	[1.90, 3.49]	[2.73, 3.90]
08/2008 - 08/2016	3.01 (51)	4.19 (18)	4.32 (12)	3.54 (15)	3.62 (13)	2.66 (13)
	[2.18, 3.83]	[2.26, 6.13]	[1.88, 6.77]	[1.75, 5.34]	[1.65, 5.59]	[1.21, 4.10]

Notes: The presented tail index values are based on the minimum $AMSE(\hat{\alpha})$ estimated through the algorithm by Beirlant et al. (1999) for each respective sub-sample. The corresponding optimal number of extremes k is shown in round brackets behind each estimate. The square brackets below each estimate indicate their 95% confidence interval.

Table 3: Tail Index Estimates based on Exponential Regressions

tends to be smaller than the left tail indices. This matches the analysis of Werner and Upper (2002) and the empirical fact, that market downturns are usually more extreme in magnitude than market booms. Consequently, the market risk of long position in these future contracts appears to be higher than of a short position.

Considering now the tail index estimates over the two individual sub-samples. In case of the Bund future, the estimate for the left tail decreases from 4.45 to 3.01 and also for the right tail from 6.01 to 4.19. The upper bound of the confidence interval indicates an unbounded fourth moment for the left tail after July 2008. Although the confidence intervals for the two periods are overlapping, the decrease in the upper bounds in the second period and the smaller confidence interval suggest a decline in the tail indices. These tail index estimates are consistent with the values of Herlemont (2005), who identifies a tail index range from three to six for daily Bund future returns from 1990 to 2004. Thus, the presented estimates indicate that the risk of extreme returns has increased since 2008. In other words, the tail risk for long position in the Bund future is higher than for a short position before and after 2008.

In contrast, the estimates for Bobl future indicate a different development over time. In case of the left tail the estimate increases from 3.81 in the first period to 4.32 in the second period and so does the confidence interval. Contrary, the estimate for the right tail decreases from 4.38 to 3.54. This indicates that before 2008 extreme losses in a long position in the Bobl future were more likely than in a short position, while after mid 2008 short positions appear to have higher tail risk. Similarly, the tail index estimates for the Schatz future increased from 2.69 to 3.62 for the left tail and decreased from

3.31 to 2.66 for the right tail. Before August 2008 the upper bounds of the confidence intervals for the left and right tail are smaller than four, indicating an unbounded fourth moment for the Schatz future during this period.

Despite for the left tail of the Bund future, the indicated development of the tail index estimates and confidence intervals is in line with the expectations based on the current monetary policy conduct. The since August 2008 implemented measures by the ECB aimed at lowering the interest rate level in the euro zone, including long-term rates, which are relevant for business investment. Consequently, the prices of German government bonds rose significantly and realized more often large positive returns than negative returns, as visualized by the price chart in Figure 1 on page 23. The increase in probability mass in the right side of the return distributions since mid 2008 is indicated by the lower estimated tail indices for all three futures.

A possible explanation for the indicated rise in left tail risk for the Bund future is the expected length of the business cycle. The Bund future price is based on the long-term yield curve up to 10.5 years. Based on past economic crises and market expectations, this time frame does not only comprise recessions but also the following economic boom, which is usually accompanied by higher inflation and a tightening of monetary policy. Therefore, at the time of introducing loose monetary policy, the expected reversal from this policy over the following five to ten years can lead to a higher expected probability of extreme negative returns in the Bund future. Since the underlying bonds of the Bobl and Schatz future do not cover the part of the yield curve beyond 2.25 and 5.5 years, respectively, their prices might not reflect the possibility of a tightening of monetary policy after this period. Thus, the risk of extreme negative returns is lower for these bonds, as indicated by the increased left tail index estimates for the Bobl and Schatz future in the second period. However, it is important to note that, despite the decreased upper bounds, the overlap of confidence intervals over time stress the need for further study in a more formal manner, as implemented in the next section.

Looking now at the relative differences in tail index estimates among the three future contracts. Regardless of the tail side and sample period, the short-term Schatz future has the lowest tail index estimates, followed by the medium-term Bobl future and then by the long-term Bund future. This is in contrast to expectations. Since the Bund future has the highest underlying bond maturity, its price is based on more information along the yield curve. This is the reason why the Bund future is used as a long-term indicator for future monetary policy. Consequently, the Bund future price contains more information, which potentially could lead to extreme price fluctuations. Similarly, the higher underlying bond duration of the Bund future, implies a higher sensitivity to changes in the yield curve and thus, a higher potential for large price changes. However, the estimates for the indices point out the reverse pattern. As in the comparison over time, the overlapping confidence intervals highlight the importance of further tests to confirm these cross-sectional differences.

A possible explanation for this ranking of tail risks might be differences in market liquidity of the futures. Looking at the descriptive statistics in Table 1 on page 22, one can see that the daily average trading volume of the Bobl future is almost half compared to the Bund future and even lower for the Schatz future.[13] The lower market liquidity implies a higher chance for market participants to influence prices by "cornering the market".[14] Consequently, the price of the short-term Schatz future might be more sensitive to large trading orders compared to the medium-term Bobl future and the highly liquid Bund future. The resulting potential for large significant price drivers might explain the fatter tails for the shorter underlying bond maturities. However, to prove the potential for market cornering in the short-term euro interest rate market, further empirical investigation is required.

It is important to note that the results presented in this analysis are solely based on the method of Beirlant et al. (1999), which induces model risk to the presented estimates. Moreover, since the confidence intervals of the tail index estimates are often overlapping in time and cross-sectional comparisons, the accuracy of presented tail index differences is significantly limited. Therefore, in order to reduce the model risk and to confirm the robustness of the presented indications, I also estimate the tail indices based on Hill plots. The results are presented in Table A.1 on page 59 in the appendix. Moreover, Figure A.1, A.2 and A.3 in the appendix show the Hill plots for the Bund, Bobl and Schatz future over the three different sample periods. For the sake of comparison to Werner and Upper (2002) and despite the mentioned shortcomings, I also report the tail index estimates based on the method of Huisman et al. (2001) in Table A.2 in the appendix. Although the estimates of the two alternative methods differ slightly in magnitude, the general indications and conclusion confirm the estimates from the Beirlant et al. (1999) algorithm. Once more, the short-term Schatz future appears to have the highest tail risks, while the long-term Bund future the lowest. Therefore, the results presented in this section, provide some first indications for time and cross-sectional differences in the tail behavior of euro interest rate future returns. However, from this it is not clarified when exactly the tail risks of the future contracts changed.

[13]From January 1999 to August 2016, the daily trading volume of the Schatz future was on 264 days less than 20% of the Bund future trading volume.

[14]One clear example of the negative relationship of extreme returns and liquidity is the Forex market. Exchange rates like EUR/USD experience very rarely large daily fluctuations.

To proof the differences over time in a more rigorous manner, I will apply the structural change test by Quintos et al. (2001) in the following section together with the proposed augmentation by Straetmans and Candelon (2013).

5.2 Testing for Structural Changes in the Tail Behavior

The results in the previous analysis give a first indication of time variation in the tail behavior of euro interest rate futures. In order to investigate these structural changes more formally and to identify potential change dates, I apply the recursive structural change test by Quintos et al. (2001), as introduced in the methodology Section 3.3. The results of the tests for the Bund, Bobl and Schatz future are presented in Table 4. The recursive test starts with a sample size of 500 observations and is increased successively by one daily observation. The supremum of the statistic for the forward version of the test Q_F indicates decreases in the tail index, while the supremum of test statistics for the backward version Q_B indicates increases in the tail index. The results are differentiated for each tail side. The optimal number of extremes in the sample k is estimated through the method by Beirlant et al. (1999) and used for the estimation of the test statistics and the bootstrap-based critical values. As shown by Straetmans and Candelon (2013), the bootstrap-based critical values apply for the forward and backward version of the recursive test. Since the bootstrapped-based critical values take into account the bias in the Hill estimator, the presented numbers in Table 4 are greater than the asymptotic critical values of 1.78 and 2.54 used by Quintos et al. (2001) and Werner and Upper (2002). This solves the problem of an over-rejection of the constant tail index null hypothesis. Moreover, the recursive test statistics of equation (3.7) and the recursive tail index estimates of equation (3.8) for the Bund, Bobl and Schatz future are plotted in Figure 2, 3 and 4, respectively. Since the recursive test starts with 500 observations, the estimates for first two years 1999 and 2000 are lost.

Considering first the results for the long-term Bund future. The null hypothesis of a constant tail index is rejected if the supremum of the test statistic is larger than the critical value at the 95% and 99% confidence level. In case of the right tail, the supremum of the forward test statistics is with a value of 20.84 significantly greater than the critical value of 5.96 at the 99% confidence level. Therefore, the null hypothesis of a constant right tail index is clearly rejected. This confirms the results from the previous analysis and implies an increase in the right tail risk of the Bund future. The time of the structural change, indicated in brackets below the test statistic, is September 26th 2008, which is eleven days after the Lehman Brothers bankruptcy. Moreover,

	k_n*		Recursive Test				Critical Values†			
	Left	Right	Left Tail		Right Tail		Left Tail		Right Tail	
			Q_F	Q_B	Q_F	Q_B	95%	99%	95%	99%
Bund	23	19	3.04	1.64	20.84***	2.66	3.22	5.98	3.12	5.96
					(26.09.08)					
Bobl	18	16	1.27	3.78**	6.95***	3.07**	3.26	6.68	2.92	6.43
				(14.02.14)	(09.09.08)	(19.07.07)				
Schatz	47	43	1.55	2.98**	2.33**	1.43	2.23	3.34	2.09	3.18
				(10.03.04)	(12.01.06)					

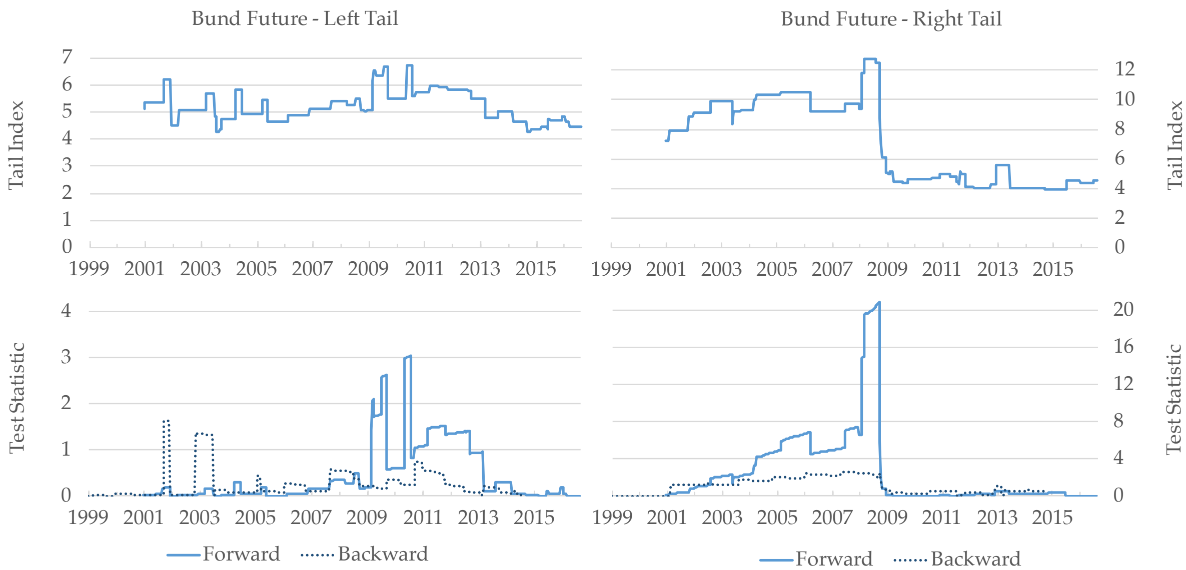
Notes: The forward and backward version of the recursive test statistic supremum are denoted by Q_F and Q_B, respectively. In case of significant breaks at the 99% confidence level, the corresponding break dates (dd.mm.yy) are reported below the test statistic.
†The critical values are based on 10,000 bootstrapped sample replications estimated with a GARCH(1,1) model for the Bund, Bobl and Schatz future, respectively.
*The values refer to the number of extremes in the full sample from 1999 to 2016, which are estimated through the Beirlant et al. (1999) method.
** Statistically significant rejections of the null hypothesis of tail index constancy at the 95% confidence level.
*** Statistically significant rejections of the null hypothesis of tail index constancy at the 99% confidence level.

Table 4: Results of the Recursive Test for Tail Index Constancy

the indicated break is just before October 2008 when the ECB started to implement its loose monetary policy by decreasing the rate on the marginal lending facility from 5.25% to 4.75% and decreasing the interest rate on the main refinancing operations from 4.25% to 3.75%. From Figure 2 it can be clearly seen that after this date the right tail index of the Bund future is constantly lower than before. The results for the backward version of the recursive test are not statistically significant for the right tail and thus, no evidence for an increase in the right tail index is found.

In contrast to the indicative results from the previous section, the null hypothesis of constant tail parameter cannot be rejected in case of the left tail of the Bund future. The suprema of the forward and backward version of the test are with 3.04 and 1.64, respectively, smaller than the 95% critical value of 3.22. This can also be seen from the left graphs in Figure 2. Compared to the right Bund tail index, the left tail index appears relatively stable over time and only fluctuates insignificantly around the value five. This results do not confirm the previous estimates, which indicate a fall in the left tail index since August 2008. Since the indicative results in Section 5.1 have overlapping confidence intervals, I regard the results of the recursive test as more reliable and conclude a constant tail parameter for the left tail of the Bund future.

Looking now at the middle-term Bobl future. In case of the right tail, the forward

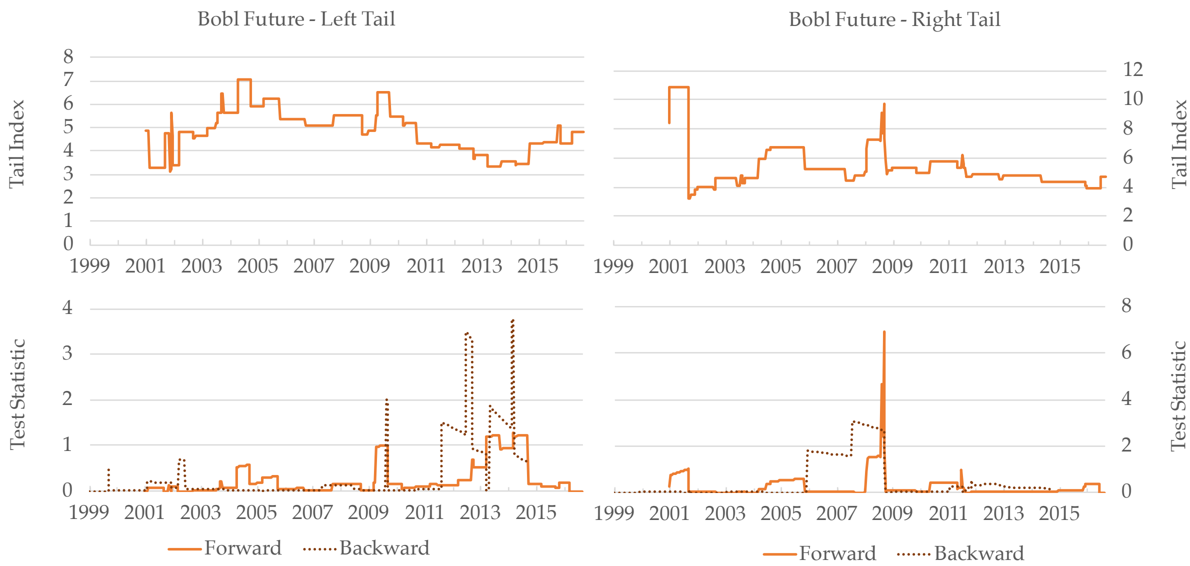


Notes: The Bund future tail index estimates in the upper part of the figure are calculated with recursive Hill estimator given by equation (3.8). The first two years of observations are lost, since the recursive Hill estimator starts with a sample size of 500 and is increasing by one daily observation. The recursive test statistic in the lower part of the figure is based on equation (3.7). The forward version of the recursive test indicates a fall in the tail index. In contrast, the backward version of the recursive test starts with the latest observation of the sample and thus, indicates a rise in the tail index. The critical values for the recursive test of the Bund future are shown in Table 4.

Figure 2: Recursive Tail Index and Test Statistic for the Bund Future

and the backward test yield significant results. The backward test indicates a rise in the tail index around July 19th 2007, significant at the 95% level. However, the forward test statistics reaches its peak with 6.95, which is greater than the 6.43 critical value at the 99% confidence level. This indicates a fall in the tail index on September 9th 2008, which is similar to the Bund future. Consequently, the probability of extreme positive returns increased during the time of Lehman Brothers' default. This confirms the indicated fall in the Hill estimates in the previous section. The increase and fall in the right tail risk of the Bobl future is also shown by Figure 3 and indicates an inverted U-shaped pattern for the tail index.

The left tail of the Bobl future shows a different behavior. The forward recursive test does not signal any statistically significant decreases in the tail index over time. However, with 95% confidence the backward test indicates an increase in the tail index around February 14th 2014. Similarly, the left graph in Figure 3 shows a slightly higher tail index after this date. However, no monetary policy change was implemented by the ECB around this time of the year. Instead, in June and September 2014 the ECB continued to decrease the interest rates on the marginal lending facility and on the main refinancing operations. Compared to the years before, 2014 represents a rather tranquil year of monetary policy changes in the euro zone. Thus, a possible explanation for the



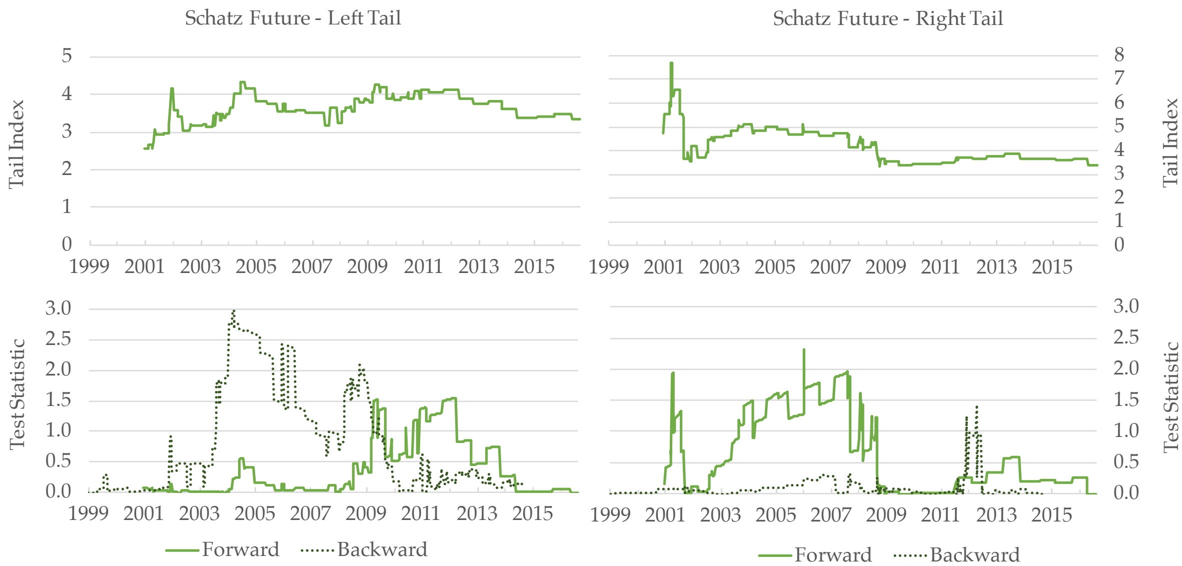
Notes: The Bobl future tail index estimates in the upper part of the figure are calculated with recursive Hill estimator given by equation (3.8). The first two years of observations are lost, since the recursive Hill estimator starts with a sample size of 500 and is increasing by one daily observation. The recursive test statistic in the lower part of the figure is based on equation (3.7). The forward version of the recursive test indicates a fall in the tail index. In contrast, the backward version of the recursive test starts with the latest observation of the sample and thus, indicates a rise in the tail index. The critical values for the recursive test of the Bobl future are shown in Table 4.

Figure 3: Recursive Tail Index and Test Statistic for the Bobl Future

decrease of left tail risk in the middle-term Bobl future is the market expectation of no reversal of the loose monetary policy conduct over the next four to five years, which would result in significant price drops of German government bonds with this maturity range. The results for the Bobl future confirm the Hill estimates from the previous section, which indicate a lower left tail risk after mid 2008.

Considering now the short-term Schatz future. In case of the right tail, the forward recursive test indicates a decrease in the tail index around January 11th 2006 at the 95% confidence level. Similarly, the upper right graph in Figure 4 shows a decreasing tail index after this date which stays at a constantly lower level from mid 2008 onwards. Again, this confirms the results presented in Table 3 in the previous section. Surprisingly, the indicated structural decrease in the tail index is more than two years before the highpoint of the financial crisis in 2008. Looking at Figure 4, the right tail index appears to reach its lower level around mid 2008 and not 2006. Since there were no major market events in January 2006, there is no obvious explanation for the decrease in the tail index at this point before the financial crisis.

For the left tail of the Schatz future the results of the backward version of the recursive test indicate an increase in the tail index over time. Around March 10th 2004 the maximum of the test statistic is with 2.98 greater than the critical value of 2.23 at



Notes: The Schatz future tail index estimates in the upper part of the figure are calculated with recursive Hill estimator given by equation (3.8). The first two years of observations are lost, since the recursive Hill estimator starts with a sample size of 500 and is increasing by one daily observation. The recursive test statistic in the lower part of the figure is based on equation (3.7). The forward version of the recursive test indicates a fall in the tail index. In contrast, the backward version of the recursive test starts with the latest observation of the sample and thus, indicates a rise in the tail index. The critical values for the recursive test of the Schatz future are shown in Table 4.

Figure 4: Recursive Tail Index and Test Statistic for the Schatz Future

95% confidence level. The permanently higher tail index after this date is also shown by the left graph of Figure 4. This increase over time was also indicated previously by the results of heuristic method in Table 3, but without a correct determination of the exact change point. Similar to the structural change in the right tail, there was no obvious major event around March 2004, which could be associated with reduced left tail risk. During the whole year of 2004 the ECB left the key interest rates unchanged. Possibly it is this very absence of extreme events or monetary policy changes, which could explain the reduced probability of the extreme negative returns for the Schatz future during this time period.

The results presented in this analysis provide convincing evidence that the probability of extreme returns of euro interest rate futures has changed since the financial crisis in 2008 and the resulting loose monetary policy conduct by the ECB. In contrast, no structural changes were found during the time of the European sovereign debt crisis. This is not surprising, since German government debt is considered as risk-free and did not realize the spike in credit spreads in 2011 as did many peripheral euro countries.[15]

[15]Schwaab and Zhang (2016) investigate the tail risk of euro peripheral government bonds during the sovereign debt crisis. The results of their novel observation-driven model indicate a reduction of tail risks through the Securities Markets Programme and Outright Monetary Transactions of the ECB.

As expected, mainly the tail index of the right side of the return distributions of the Bund, Bobl and Schatz future decreased during the course of the financial crisis. Comparing the tail index estimates and test statistics of the three futures in Figure 2, 3 and 4, it appears that the Bund future has experienced the most significant decrease in the right tail index, which matches expectations. Werner and Upper (2002) do also find time variation in the tail behavior of the Bund future, however, only at high frequencies and not in daily returns. However, their sample period from 1997 to 2001 does not include the extreme events of the global financial crisis and the European sovereign debt crisis, which have the potential to lead to large daily price fluctuations in the financial markets.

The presented results also have implications for the margin requirements of these euro interest rate futures. Naturally, due to the highest tail risk in the Schatz future, one would expect the highest initial margin for the short-term future. However, the initial margin requirements for the Bund, Bobl and Schatz future are 1.61%, 0.78% and 0.19% of the executed future price, respectively.[16] These initials margins are set by Eurex and updated daily. The margins are based on two components, which are market risk and liquidity risk. Here, the market risk component is determined through tail risk measures in form of value-at-risk figures. Although the Schatz future has historically the thickest tails, relatively to the Bund and Bobl future, its value-at-risk estimates are significantly smaller. The reason for this can be seen from the historical maximum and minimum returns in Table 1 and from Figure 1 on page 23. The extreme returns of the Schatz future are significantly smaller in absolute terms than for the other two futures. The same applies to the medium-term Bobl future compared to the long-term Bund future. The absolute magnitudes of these returns are translated into their value-at-risk estimates. Consequently, the value-at-risk estimates for the Schatz future will be lower than for the Bund and Bobl future and therefore, also the initial margin requirement. Similarly, the value-at risk-estimates and margin requirements of the medium-term Bobl future are smaller than for the Bund future, but larger than for the Schatz future.

Nevertheless, the presented estimates in this section indicate significant differences in tail risk between the left and right side of the return distributions of all three future contracts. Consequently, the risk of short position does not equal the risk of a long position in the future. Since the initial margin required by Eurex is identical for long and short positions, the implied VaR estimates do not differentiate between the tail risk for left and right side of the distribution i.e. they assume symmetry. This represents a cost disadvantage for the position with the lower tail risk. Since this risk is also not

[16] These margin requirements were set by the European Exchange (Eurex) on December 14th 2016.

constant over time, a fair risk assessment requires a daily update of differentiated initial margin requirements for long and short position in the Bund, Bobl and Schatz future. In order to investigate this issue further, the following Section 6 presents a detailed analysis of differences in tail risks and the resulting value-at-risk estimates for euro interest rates futures.

The presented results have another important implication for the margin requirements in terms of profitability. As shown by this analysis, the tail risk of a position in the Bund, Bobl and Schatz future has increased since the massive market interventions by the ECB as a response to the financial crisis and sovereign debt crisis. Consequently, the margin requirements reflect the increased probability of extreme price fluctuations by higher initial margins. These higher margin requirements set by the exchange reduce leverage and equal stricter capital requirements, which are costly for banks and reduce the profitability of their trading activities.[17] At the same time, European banks are challenged by additional capital requirements from the regulatory side. If the monetary policy conduct by the ECB is the cause of higher tail risk in the euro interest rate market and thus, the reason for higher initial margin requirements, then this would constitute a double encumbrance for banks created by the authorities. Consequently, additional to reduced interest rate margins, banks would have a further argument against the current expansionary monetary policy of the ECB or alternatively against stricter capital requirements. However, this argument requires further research and establishing a causal link. In the following section I investigate if the tail index contains information beyond the concept of volatility and how different value-at-risk estimates for the Bund, Bobl and Schatz future developed since the implementation of the expansionary monetary policy by the ECB.

6 The Tail Index and Other Market Risk Measures

The third component of the analysis presented in this paper is to investigate if the tail index adds information to the conventional market risk measure volatility. As shown in Section 5.2, there has been a structural change in the tail risks of the Bund, Bobl and Schatz future. Consequently, the probability of extreme returns of a position in these future contracts has changed. I study if the volatility captures this change in risk

[17] Since 2009, trading activities by banks have been massively reduced as response to higher capital requirements and reduced profitably. Moreover, since the introduction of the Volcker Rule in 2010 proprietary trading by commercial banks is prohibited.

	Volatility		Tail Index Left		Tail Index Right	
Structural Change Date	$\hat{\sigma}_1$	$\hat{\sigma}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
Bobl						
09.09.08	0.217	0.243	3.81 (21)	4.32 (12)	4.26 (95)	3.54 (15)
Bund						
26.09.08	0.322	0.401	5.15 (12)	3.04 (50)	6.03 (31)	4.32 (17)

Notes: The presented tail indices $\hat{\alpha}_{1,2}$ and realized volatilities $\hat{\sigma}_{1,2}$ are based on the two sub-periods before and after the identified structural breaks in Section 5.2. Only structural changes significant at the 1% level are considered. The presented tail index values are based on the Hill estimator equation (3.2) and the minimum $AMSE(\hat{\alpha})$ estimated through the algorithm of Beirlant et al. (1999) for each respective sub-sample. The corresponding optimal number of extremes k is shown in round brackets behind each estimate. The volatilities are calculated according to equation (6.1).

Table 5: Comparison of the Tail Index and Daily Realized Volatility

over time. Moreover, I investigate the implications of an inaccurate consideration of tail events, in terms of value-at-risk estimates for different long and short positions.

6.1 The Tail Index and Realized Volatility

In this analysis I compare the tail index estimates with the realized volatility of future returns before and after structural change dates, which were identified in Section 5.2 and are significant at the 1% level. For this I split the full sample into two sub-samples according to the identified structural change date. Since there is no evidence for a structural change in case of the short-term Schatz future at the 1% significance level, I only consider the cases of the Bund and Bobl future in September 2008. I calculate the daily realized volatilities according to the following formula:

$$\hat{\sigma}_t = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n} {r_t}^2} \tag{6.1}$$

where $n$is the sample size and r_t is equal to the daily return observations. By setting the mean equal to zero, equation (6.1) yields an estimator which is independent of a trend in the return series. However, even with the incorporation of the actual means the results would differ only marginally, as can be seen from the very small mean returns presented in the descriptive statistics in Table 1 on page 22.

The realized volatilities of the Bund and Bobl future before and after significant change dates are shown in Table 5. Moreover, the table includes the tail index estimates for the two sub-samples. The tail indices are estimated according to the methodology of

Beirlant et al. (1999). In contrast to the analysis of Werner and Upper (2002), I do not only compare the realized volatility to the left tail but also to the right tail of the return distribution. Thereby, the tail risk of long and short position in the future contracts can be differentiated. The first significant structural change in the tail behavior occurred in case of the right tail of the Bobl future on September 9th 2008. Before this day the daily realized volatility over the whole sub-sample is equal to 0.217%. After this date and until August 2016 the daily realized volatility increased to 0.243%. This matches the decrease of the tail index in case of the right tail of the Bobl future from 4.26 to 3.54. However, the tail index estimate for the left side of the distribution is equal to 3.81 in the first period and 4.32 in the second period. Thus, the tail index estimates indicate that the probability of extreme negative returns is lower after September 9th 2008, while the volatility actually indicates an increase in market risk. This demonstrates an information asymmetry between the tail index and the volatility and also highlights the importance of a differentiated analysis of the left and right side of the Bobl future's return distribution.

The second significant structural change in the tail behavior occurred in the case of the right tail of the Bund future on September 26th 2008. Before this date, the tail index is estimated at 6.03 and at 4.32 the period after. This indicated probability increase of extreme positive returns is also captured in the realized daily volatility, which increases from 0.322 to 0.401. Similarly, the left tail index of the Bund future decreases from 5.15 to 3.04 after the indicated structural change date. In this case the higher implied probability of extreme negative and positive returns can be inferred correctly from the higher volatility. Therefore, over the considered time period and in contrast to the Bobl future, no additional risk information can be retrieved from the tail index which is not contained in the volatility.

The results for the Bund future do not match the analysis of Werner and Upper (2002). However, since the two authors use intra-day 5 minute returns for the Bund future, there could be information differences in the frequency of data. Nevertheless, the results for the Bobl future are consistent with Werner and Upper (2002) and indicate that the assessment of market risk exclusively by volatility could lead to a serious misjudgment of the possibility of extreme events and thus, to large unexpected losses for market participants. In the following section I investigate this topic further in terms of daily value-at-risk estimates for the Bund, Bobl and Schatz future.

6.2 Implications for Value-at-Risk Estimates

Additional to volatility, financial institutions commonly quantify market risk by the value-at-risk (VaR) method. Due to its simple interpretation of only one number, it is easily understood and can be translated into an actual money amount. Furthermore, its wide acceptance in the industry also allows for comparison between different assets and investment strategies. In general, the value-at-risk of an asset is implicitly given by the exceedance probability:

$$P(X > VaR) = p$$

Through rearranging the VaR is simply determined by the p-quantile of the loss distribution:

$$VaR = F^{-1}(1 - p)$$

Depending on the distribution function, the calculated VaR indicates the maximum loss expected to occur over a target time horizon at a pre-specified confidence level. However, it does not represent a loss forecast or the worst possible scenario, since there is a low pre-specified probability that the actual loss will exceed the VaR estimate.

As proven in the previous section, there has been a change in the probability of extreme returns in the euro interest rate market over the past years. Naturally, the question arises if this change in tail risk is incorporated accurately in the conventional value-at-risk estimation methods, which will be investigated in this section. In this analysis I do not specify the amount invested in the Bund, Bobl or Schatz future contracts. For my calculations the VaR simply represents a daily percentage i.e. daily return. This enables risk comparisons independent of the contract size and invested amount. It should be noted that in case of futures the actual amount invested is based on the specific margin requirements and not equal to the contract price or size of the future. Nevertheless, due to the leverage, the VaR usually refers to the amount of the contract price. Moreover, since I investigate and compare both sides of the return distribution, I always report VaR as a positive percentage for each tail side. This allows to study conveniently the risk of both long and short positions in the future contracts. In the following section I explain the different VaR methods, which are applied in the analysis.

6.2.1 Applied VaR Estimation Methods

In general, there are several methods to calculate the value-at-risk of financial securities. In the following I focus on the empirical and the normal parametric approach. I compare their estimates with the results of a quantile method, which takes into account the tail index. Since there are several versions of these approaches, I explain the methodology I apply. In case of the empirical approach the VaR estimate is based on a ranking of the historical returns and on a critical observation according to a specified confidence level. I calculate the empirical VaR according to the following formula:

$$Empirical\, VaR = exp(R^*_\alpha) - exp(\mu) \tag{6.2}$$

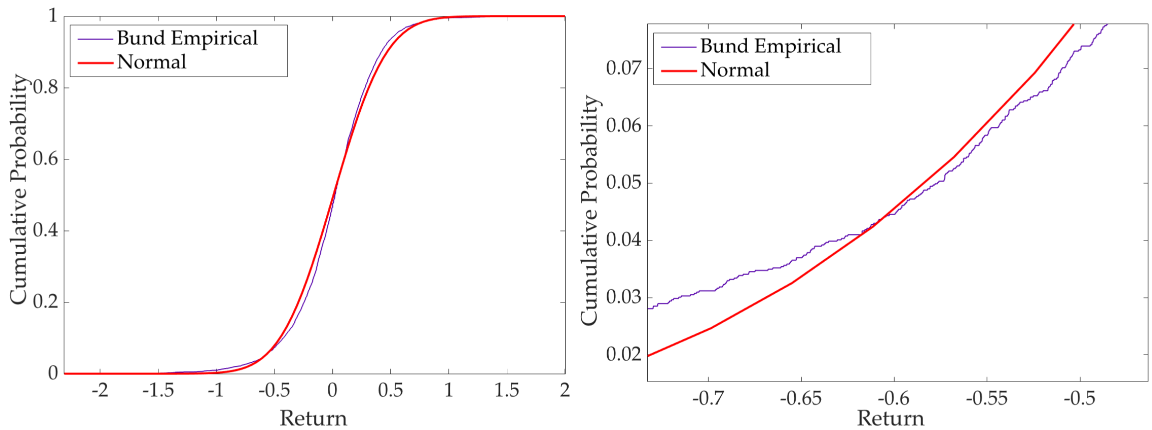
where R^* equals the return observation at the α percentile of the respective sample and μ is equal to the sample's mean return.[18] A major weakness of this method is its assumption of a constant return distribution. A historical absence of extreme jumps is not a guarantee for their non-occurrence in the future. Furthermore, the VaR forecasts are dependent on discrete historical extremes. This implies that the forecasts are also non-continuous and the VaR might underestimate or overestimate the true risk. The right part of Figure 5 visualizes this problem by the example of the Bund future. The empirical cumulative probability function in the right graph of the figure is clearly a non-smooth curve, which reduces the accuracy of VaR estimates. Despite these disadvantages, financial institutions often prefer this non-parametric method, since it does not require frequent changes of trading limits and costly capital adjustments. Especially for highly diversified portfolios with a large number of different assets, the empirical VaR does not require constructing and updating large variance-covariance matrices (Danielsson, 2000).

In the case of the parametric approach, I assume returns follow a normal distribution.[19] Based on the central limit theorem and the law of large numbers it is consistent with Markowitz' modern portfolio theory and Black-Scholes option-pricing. The volatility is defined by historical returns and assumed to be an accurate risk predictor for the future. This assumption is supported by the fact that returns exhibit dependence in the second moment. I calculate the normal parametric value-at-risk by the following equation:

$$Parametric\, VaR = \sigma \times z_\alpha \tag{6.3}$$

[18] It is important to note that the α percentile is not to be confused with the tail index. In existing literature both variables are notationed with the same Greek letter, which is also applied in this paper.

[19] J.P. Morgan RiskMetrics uses a parametric approach with IGARCH models and normal innovations (Danielsson and de Vries, 1997).



Notes: The left graph shows the cumulative probability of the normal distribution and of the empirical distribution of daily Bund future returns. The right graph shows an enlargement of these two cumulative probabilities. The empirical distribution is based on the time period from January 4th 1999 to August 22nd 2016.

Figure 5: Cumulative Distribution of Daily Bund Future Returns

where σ is the standard deviation i.e. volatility of the returns series over the respective sample. The critical value z_α can be taken from the cumulative standardized normal distribution. A major disadvantage of this approach is the underestimation of extreme returns. In case of the Bund future this can be seen from the lower normal cumulative probability relative to the empirical in the right graph of Figure 5. The implied negligence of fat tails in the normal distribution allows me to illustrate the severeness of an underestimation of extreme events in terms of market risk for euro interest rate futures. In practice, the volatility smile in option pricing confirms that financial markets do account for the significant probability of tail events in the prices of these insurance products against market risks.

One solution to this shortcoming of the normality assumption is a widely applied modified version of the parametric approach. This modified VaR is based on a more precise characterization of the return distribution by taking into account the third and fourth moment of the return series. The estimation is similar to the normal parametric VaR:

$$MVaR = z_\alpha^{CF} \times \sigma \tag{6.4}$$

However, in this case the critical value z_α is adjusted for the skewness and kurtosis through the Corner-Fisher expansion:

$$z_\alpha^{CF} = z_\alpha + skew \times \left(\frac{z_\alpha^2 - 1}{6}\right) + kurt \times \left(\frac{z_\alpha^3 - 3z_\alpha}{24}\right) + skew^2 \times \left(\frac{2z_\alpha^3 - 5z_\alpha}{36}\right) \tag{6.5}$$

Usually the kurtosis of a return series has the largest effect on the augmented critical value. Similar to the normal parametric VaR, the modified VaR applies to both sides of the return distribution.

Since the empirical, normal parametric and modified methods are based on different assumptions with distinctive advantages and disadvantages, I will compare each of them to a value-at-risk estimate, which incorporates the tail index explicitly. Schmid and Trede (2006) demonstrate how to determine these VaR estimates through extreme quantiles based on Hill-estimations of the tail index α. Assuming that far in the tails the distribution behaves like a Pareto distribution, the Hill-estimated probability of an extreme event $P(X > x)$ is equal to:

$$\bar{F}_{n,Hill(x)} = \hat{P}(X > x) = \frac{\hat{k}}{n} \times \left(\frac{x_{(n-k+1)}}{x}\right)^{\hat{\alpha}_{Hill}}$$

In this case I am interested in large positive returns x and the estimation of the p-quantile x_p with p close to the value one. Through the inversion of $\hat{P}(X > x)$ and taking into account that $\hat{P}(X > x) = 1 - p$, I can calculate the Hill-estimated value $\hat{x}_{p,Hill}$ for an extreme p-quantile x_p:

$$\hat{x}_{p,Hill} = x_{(n-k+1)} \times \left(\frac{n}{\hat{k}} \times (1-p)\right)^{\frac{-1}{\hat{\alpha}_{Hill}}} \tag{6.6}$$

Equation (6.6) renders a value-at-risk estimate, which takes into account the tail risk of a return series through the Hill estimate $\hat{\alpha}_{Hill}$ for the tail index α. This approach is identical to the semi-parametric quantile estimator by de Haan et al. (1994). It allows for a differentiated analysis of the tail risk of both sides of the return distribution. I determine the optimal number of extremes in the sample k through the method by Beirlant et al. (1999) for each tail side of the return distributions of the Bund, Bobl and Schatz future. Additionally, to determine the VaR for the left side of the distribution with equation (6.6), I simply reverse the signs of the return series. The right tail describes then negative returns, which reduces the computational burden significantly.

In this analysis I calculate rolling VaR estimates for both tail sides of the Bund, Bobl, Schatz future based on the empirical (6.2), normal parametric (6.3) and its modified approach (6.4). Moreover, I calculate rolling Hill-estimated extreme quantiles (6.6), to which I refer as tail index VaRs in the following. These estimates use a rolling sample window of two years i.e. 500 daily observations starting in January 1999.[20] For the extreme quantiles (6.6) I calculate rolling Hill estimates $\hat{\alpha}_{Hill,t}$ with a sample window

[20] I assume 250 trading days per calendar year.

of 500 observations. These Hill estimates are based on a constant number of extremes in the tail $\hat{k}$, which is derived from the analysis of Straetmans and Candelon (2013). In this case, $\hat{k} = \hat{c}t^{\frac{2}{3}}$ with $t = 500$ and $\hat{c} = \frac{\hat{k}_n}{n^{\frac{2}{3}}}$, where n is the full sample size for the respective left and right tail of the future's return distribution. Since $\hat{c}$ and t are constants, $\hat{k}$ is also constant through time. The number of extremes in the full sample $\hat{k}_n$ is determined according to the algorithm by Beirlant et al. (1999). The number of extremes $\hat{k}$ are estimated separately for the left and right tails of each Bund, Bobl and Schatz future and also used for the k parameter in equation (6.6).

For effective risk management it is recommended to look deep into the distribution's tail and use a critical level of 0.1%.[21] This corresponds to an event expected to occurs every 1,000 days or 3.97 years. However, since my VaR estimates use a rolling sample size of 500 observations, I use the 1% critical level. The aim of this analysis is to identify differences among the four VaR methods in case the Bund, Bobl and Schatz future. Consequently, the choice of the critical level does not influence the analysis' outcome, as long as it is chosen consistently among all methods. This also applies to the size of the sample. Generally larger sample sizes are preferred. However, by increasing the number of observations in a rolling sample, risk estimates like the volatility smooth out, since single extreme returns account relatively less in larger samples. Thus, due to the information loss about temporary market risks, increasing the sample size above 500 observations is not advantageous for the purpose of this analysis.[22]

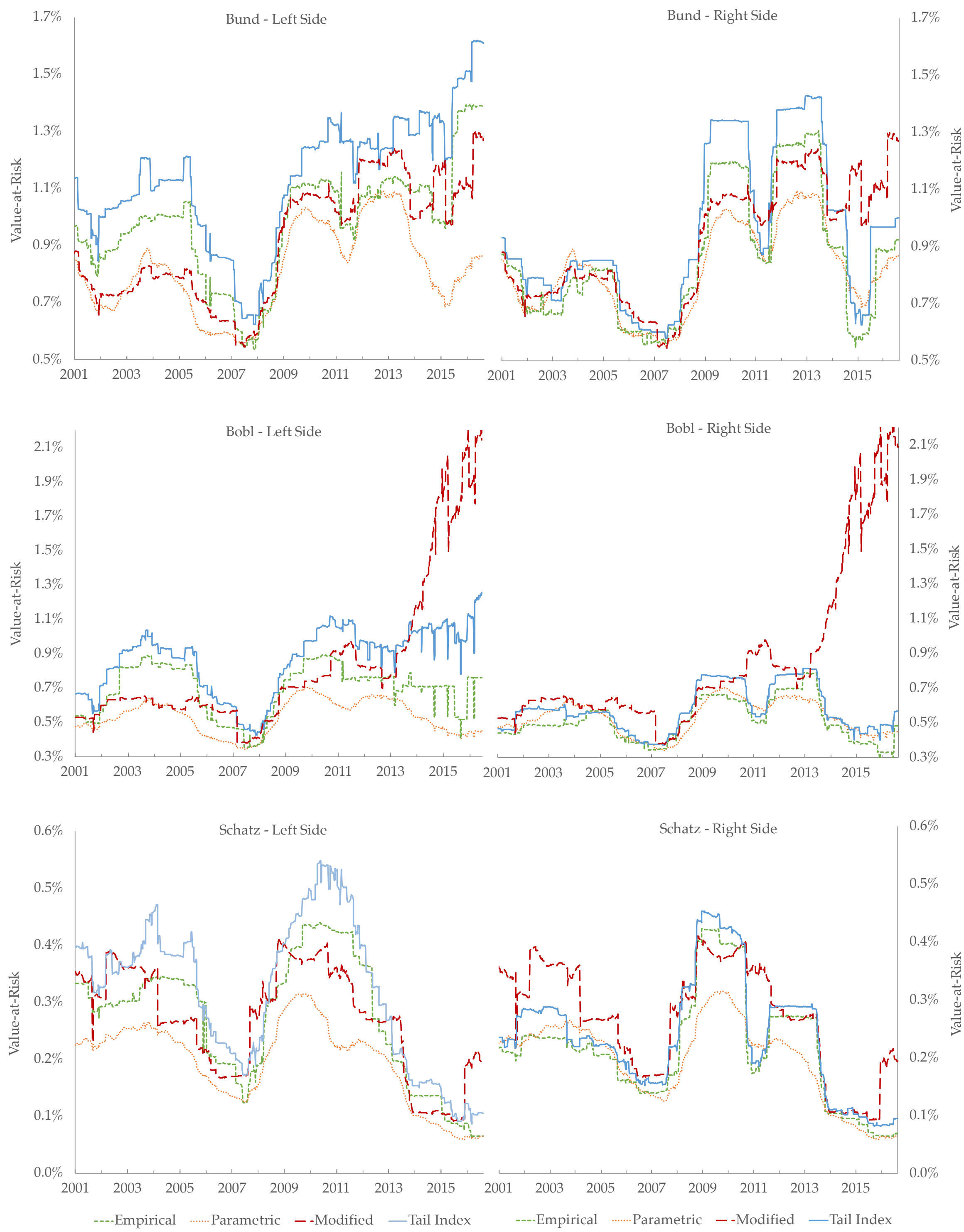
6.2.2 Value-at-Risk Estimates for Euro Interest Rate Futures

In this section I investigate if any risk information advantage arises through the inclusion of the tail index in value-at-risk considerations in the case of euro interest rate futures. Thereby, I can assess if the tail index contributes to an enhancement of market risk management for different long and short positions in the future contracts. The rolling VaR estimates based on the empirical, normal parametric and modified method are presented in Figure 6 as well as the rolling Hill-estimated extreme quantiles.

In case of the left tail of the long-term Bund future, there has been a significant increase in all VaR estimates since 2008. Surprisingly, all methods indicate low market

[21] The Basel Committee on Banking Supervision usually the requires value-at-risk calculations to be based on a critical level of 1%. Such returns are expected to occur approximately every 100 trading days and do not necessarily represent extreme events.

[22] In order to confirm the robustness of the results to the choice of the critical value and the sample size, I re-estimate the VaR models with a critical level of 0.1% and a sample size of 1000 observations. The level of the VaR estimate increases due to the lower critical level, but the main conclusion of the analysis remains robust. For the sake of brevity, the numerical results are not displayed in this paper, but available from the author upon request.



Notes: The left side of the figure refers to negative returns, while the right side applies to positive returns. The empirical, parametric, modified and tail-index VaRs are calculated according to equations (6.2), (6.3), (6.4) and (6.6), respectively. The rolling window size t is held constant over time at 500 observations. The tail index VaR is based on a constant k over time and estimated for each respective return series and tail side.

Figure 6: Rolling Daily Value-at-Risk Estimates at the 1% Critical Level

risk just before the financial crisis, which possibly resulted in unexpected large losses for traders in the following months. The Hill-estimated quantile renders constantly the highest VaR estimate over the whole sample. Since 2008 this estimate is steadily increasing and reaches its peak in 2016. The empirical and modified VaR indicate a similar increase over the past eight years, although most of the time at a lower level. In contrast, the parametric VaR does not capture the risk increase since 2013 and actually declined to a lower level, which could lead to fatal underestimation of the possibility of extreme negative returns in case of the Bund future. In general, the parametric VaR indicates lower market risk than the other methods. This follows from the fact that the normal parametric approach does not account for the possibility of tail events sufficiently, which leads to an underestimation of extreme returns. This confirms the results of Herlemont (2005), who conducts a similar study for the Bund future over a period from 1990 to 2004. The immense difference between the normal parametric estimate and the Hill-estimated quantile illustrates the usefulness of the tail index in value-at-risk considerations and thus, in market risk applications.

Looking at the right tail of the Bund future, the differences between the methods appear to be smaller, especially before the financial crisis in 2008. However, the relative levels of VaR estimates are similar to the left tail and dominated by the tail index VaR until 2014. In contrast to the left tail, the empirical and tail index VaR confirm the lower market risk since 2013, which is signaled from the parametric method. This highlights the problematic of the symmetric parametric VaR and its modified version. As the other methods confirm, the decreased market risk since 2013 does only apply to short positions in the Bund future and not to long positions. Therefore, an undifferentiated analysis of tail risks leads to a fatal underestimation of the possibility of extreme price drops of the Bund future. The differences in value-at-risk estimates for the left and right tail also stress the importance of different margin requirements for long and short positions.

Important to note from Figure 6 are also the two plateaus of market risk in the right tail of the Bund future, which are indicated by all four VaR methods. The first plateau occurred when the ECB reacted to the financial crisis and its aftermath around 2009 and 2010. During this period the ECB continued to lower key interest rates and introduced unconventional measures like longer-term refinancing operations (LTRO) or the Securities Markets Programme (SMP). The resulting lower interest rate level drove up bond prices and led to large positive returns for existing bond holders. Consequently, this risk plateau is mainly visible in the right side of the return distribution. The parametric VaR and its modified version also indicate incorrectly these plateaus for the left side, because they are unable to differentiate the sides of the return distribution. The second plateau starts in 2011, the time of the European sovereign debt crisis. During this period the interest rate differentials of euro peripheral and German government bonds, known as credit spreads, increased dramatically, which caused massive speculation about a breakdown of the whole euro system. In order to prevent worse, the ECB had to intervene and provided even more liquidity to the markets. Surprisingly, the level of market risk for the Bund future appears to be very similar during the financial crisis and sovereign debt crisis, although German government debt is considered as risk-free and thus, should be less effected by the latter crisis.

Furthermore, after a period of relatively low risk in 2014, all methods indicate a considerable rise in risk in the right tail of Bund future since the beginning of 2015. In January 2015 the ECB announced its quantitative easing program and implemented it in March the same year. Therefore, the higher VaR estimates might indicate that the unconventional policy measures by the ECB can also have destabilizing effects and lead to higher risk in the euro interest rate market. As argued in the previous section, this translates into higher initial margin requirements for the future contracts and thus, reduces the profitability of trading activities. Additionally, looking at the dates of extreme price movements of the Bund future since January 2015, they appear to be consistent to a large extent with ECB press conference dates.[23] However, large price fluctuations did also occur during conferences in which no monetary policy change was announced, although largely expected by the markets as indicated by Bloomberg consensus estimates. Thus, through the massive intervention in the financial markets, the ECB might also has created a basis for speculation about future monetary policy and resulting asset prices. This indicated increase in market risk represents an argument against the continuation of the loose monetary policy conduct by the ECB.

Analyzing now the VaR estimates for the left tail of medium-term Bobl future. Most notably the modified VaR significantly increased from 2014 onwards and reached a level almost twice as high as the tail index VaR. This dramatic rise is driven by the increase in the kurtosis value from 12.01 to 35.26. It is important to realize that the reported estimates are based on the 1% critical level. Thus, these returns are expected to occur every 100 trading days. The most recent modified VaR estimate for the left tail of the Bobl future is 2.09%. However, looking at the descriptive statistics in Table 1, one can see that over the past 4490 trading days (17 years) the most negative return was only 1.72%. Consequently, this raises serious doubts about the accuracy of the modified VaR

[23]For an empirical analysis of which news are affecting the euro bond market, see Andersson et al. (2009) or Asplund (2011).

estimate. The problem of this risk overestimation is even more severe in case of the right tail of the Bobl future. While all other methods indicate a fall in market risk since 2013, the modified VaR signals the opposite in an extreme magnitude. This can result in higher capital requirements, which are costly for financial institutions. Therefore, non-symmetric VaR estimations, as in the empirical and tail index approach, are clearly preferred in this case.

Considering the right tail of the Bobl future, the two risk plateaus around the financial crisis and the European sovereign debt crisis are indicated by all methods, despite the modified VaR. However, the level of these plateaus appears relatively smaller in the large scaling of the graph due to the high modified VaR estimates. In contrast to the Bund future, the VaR estimates for right tail of the Bobl future did not rise significantly since January 2015. The reason for this might be that many German government bonds with shorter maturities are not eligible for the quantitative easing program, since the yield on these bonds is below the deposit rate and would result in direct losses for the ECB.[24] Consequently, these bonds are not bought on a large scale by the ECB and do not realize the extreme positive returns as their long-term equivalent. These results of the Bobl future confirm the previous Bund future findings and show that the symmetric parametric approach and its modified version are inappropriate risk measures for these assets, especially in the current market environment.

Regarding the short-term Schatz future, the results for the normal parametric and its modified approach look more promising. In both tail sides the direction of the different VaR estimates is largely consistent over time. Again, the two market risk plateaus in the right tail around the financial crisis and the European sovereign debt crisis are clearly visible among all VaR methods. However, in this case the financial crisis appears to be more severe in market risk than the sovereign debt crisis. Furthermore, all methods signal decreasing left tail risk since 2010. This contrast to the Bund and Bobl future reveals that it is dangerous to infer the market risk of a future position from another future with a different underlying maturity. Figure 6 visualizes that the VaR estimates for the Bund, Bobl and Schatz future do not always evolve identically over time, remarkably in recent years.

As indicated in Section 5.1, a possible explanation for this might be the expected length of the business cycle. Since the underlying bonds of the Schatz future do not cover the part of the yield curve beyond 2.25 years, their prices might not reflect the possibility of a tightening of monetary policy after this period. Thus, the risk of extreme

[24] As of November 15th 2016, the yield on German government bonds with a remaining maturity of eight years or less is negative.

negative returns is lower for these bonds, as indicated by the decreasing VaR estimate. To investigate this point further I calculate rolling yearly correlations between the three future returns. The results are presented Figure A.4 on page 58 in the appendix. From the figure can be seen that the correlations between the short-term Schatz future and the two other futures significantly dropped in 2014. This indicates a non-parallel shift in the euro yield curve. Although slightly recovering, the non-constant correlations needs to be taken into account especially in the risk assessment of large interest rate portfolios.

Three aspects are important to take away from this analysis. First, the necessity of a differentiated consideration of tail risks in the left and right side of the return distribution has grown since the financial crisis and the resulting market intervention by the ECB. The divergent VaR estimates indicate significant differences in tail risks for long and short positions in the Bund, Bobl and Schatz future. Symmetric measures, like the normal parametric and its modified version based on the Corner-Fisher expansion, lead to a fatal inaccurate assessment of market risks. The Hill-estimated extreme quantile overcomes this shortcoming and takes into account the tail index for each distribution side separately. Although the empirical VaR follows the tail index VaR closely, the latter yields more conservative estimates and thus, should generally be preferred among all four methods. Second, as shown by the non-constant return correlations, the risk of a specific euro interest rate future is not an accurate indicator for the market risk of another future with a different underlying bond maturity. Consequently, for effective market risk management it is not only essential to differentiate the left and right tail risks, but also to account for differences in market risks along the yield curve. Third, together with the results in the previous sections, this analysis indicates that the actions by the ECB might also lead to higher risks in the euro interest rate market. For a more formal argument against an extensions of the current monetary policy conduct further research is recommended.

7 Conclusion

Since the financial crisis in 2007/08 and the European sovereign debt crisis, the ECB has conducted expansionary monetary policy measures on an unprecedented scale. These actions resulted in a historical low interest rate environment in the euro zone, which inflated bond prices and led to a more risk-taking behavior in the search for yield in the euro interest rate market. One of the most relevant securities in this market are futures on German government bonds, namely the long-term Bund, the medium-term Bobl and the short-term Schatz future. The underlying German government bonds are considered as risk-free and only reflect the general interest rate level. Therefore, these futures are widely used as a monetary policy indicator and main hedging instrument for euro interest rate transactions like swaps. In this paper I investigate time and cross-sectional differences in the tail behavior of the Bund, Bobl and Schatz future during the financial crisis and the sovereign debt crisis.

The analysis comprises three aspects. First, I investigate over the period from 1999 to 2016 if the daily returns of Bund, Bobl and Schatz futures exhibit fat tails and if there are significant differences among these futures with respect to tail risks. Here, the variable of main interest is the tail index α, which measures the speed at which the mass in the tail decays as one looks deeper into the tail. Since these three futures have different underlying bond maturities and durations, I hypothesize that the long-term Bund future has the highest tail risk, followed by the medium-term Bobl and then by short-term Schatz future.

Second, I analyze if the tail risk of the future contracts has significantly changed over the period from 1999 to 2016. As a response to the financial crisis and the European sovereign debt crisis, the ECB intervened heavily in the financial markets by suppling immense liquidity and lowering interest rates. Therefore, I hypothesize a structural change in the tail indices around 2008, which should indicate an increase in the probability of extreme returns for the Bund, Bobl and Schatz future. Since the current ECB policy conduct aims at lower interest rates, I expect that only the right tail of the bond futures' return distribution indicates the increase in risks since 2008. Additionally, due to the longest underlying bond maturity, I anticipate that the structural change in the Bund future is most significant.

In the third aspect of this paper, I investigate the relation of the tail index to alternative market risk measures. I analyze if the tail index contains information beyond the realized volatilities of the Bund, Bobl and Schatz future. Since volatility is applied as a symmetrical market risk measure for long and short positions, I anticipate a significant misjudgment of market risks for different positions in these future contracts. Moreover, I study the implications of the tail index in terms of value-at-risk considerations. For this I compare VaR estimates of conventional methods to an alternative estimation approach, which takes into account explicitly the tail index in the calculation process. The outcome I presume is not only a more conservative VaR estimate, but also a more accurate market risk differentiation for long and short positions in euro interest rate futures.

Anticipating the results, I find significant evidence for fat tails in the daily return distribution of the Bund, Bobl and Schatz future over the considered time period. Measured by the tail index and regardless of the return distribution side, the short-term Schatz future appears to have the highest tail risks, followed by the medium-term Bobl future and then by the long-term Bund future. This is in contrast to the expectations based on the underlying bond maturities and durations. A significantly lower level of market liquidity, relative to the other two future contracts, might be an explanation for the highest tail risk of the Schatz future.

The second part of analysis provides evidence for structural changes in the tail indices of the Bund and Bobl future during the financial crisis in 2008. In line with the hypothesis, the structural change in tail indices during this crisis only applies to the right tail of the return distributions and is most significant for the long-term Bund future. In case of the Schatz future the results indicate higher risk in the right tail already before the financial crisis. However, also evidence is found for a structural decrease in left tail risk for the Bobl and Schatz future in 2014 and 2004, respectively, although with lower significance. Compared to previous years, the absence of drastic monetary policy changes might be an explanation for this indicated lower risk in the left tail.

The results for the third component of the study reveal that tail index contains risk information, which is not captured by the realized volatility of the future contracts. As the analysis shows, this has also significant implications for the market risk management of financial institution in value-at-risk considerations. By explicitly taking into account the tail index, the accuracy of value-at-risk estimates for different long and short positions in euro interest rate futures can be significantly improved. Especially in the current market environment, this highlights the problematic of symmetric market risk assessments in case of the Bund, Bobl and Schatz future. Moreover, the results indicate that the recent unconventional monetary measures by the ECB led to an increase in tail risks in the euro interest rate market, which raises the capital costs for financial institutions through higher margin requirements. Consequently, this demonstrates a controversy of an extension of the ECB's current policy conduct. However, for a more formal argument further research is required.

The results of this study contribute to the existing literature in several ways. To best of my knowledge, I am the first one who investigates explicitly the tail behavior of the Bund, Bobl and Schatz future over the course of the financial crisis and the European sovereign debt crisis with the resulting ECB policy conduct. Furthermore, the direct comparison of the tail risks of these three future contracts over the period from 1999 to 2016 is the first of its kind. Since the full consequences of the present unconventional monetary policy of the ECB are still unknown, this paper contributes to a better understanding of the resulting extreme risks in the euro interest rate market. The existing literature primarily studies the long-term Bund future and considers data before the financial crisis in 2008. The research conducted by Werner and Upper (2002) concerning the tail behavior of the Bund future represents the basis of the analysis presented in this paper. My estimated values for the tail indices of the Bund future are largely in line with previous estimates of existing research. However, Straetmans and Candelon (2013) prove shortcomings in the approach chosen by several studies existing in this field. Using the results of the two authors, I overcome these deficiencies in this paper and thereby, improve the accuracy and robustness of the findings.

Nevertheless, the limitations of the results presented in this paper stress the importance of further research in this area. The different methods for determining the optimal number of extremes in the tail indicate the same results for cross-sectional differences in tail risks for the Bund, Bobl and Schatz future. However, the lack of a generally accepted approach highlights the possibility of estimation errors. Moreover, as Werner and Upper (2002) show in their analysis, differences in tail risks in the frequencies of return data exist. Consequently, the findings in this paper might not hold for intra-day returns. Since the impact of extreme events is likely to diminish over time, an extension of the analysis to intra-day data for the Bund, Bobl and Schatz future could contribute to a better understanding their tail behavior.

Additionally, for risk management applications the analysis should be extended to scenarios in which the VaR is exceeded. Since such events can lead to immense distress for financial institutions overnight, the estimation of the expected shortfall appears to be of great relevance for the industry. The results indicate that the Hill-estimated extreme quantile yields relatively conservative market risk estimates. However, formal backtests of the VaR figures should give more insights into their actual effectiveness in market risk management. Moreover, the theory of liquidity differences as a driving force of cross-sectional variation in tail risks of the Bund, Bobl and Schatz future requires further investigation. In general, the development of an statistical model, which contributes to a better understanding of the return variance in the euro interest rate market, appears to be an interesting challenge for future research.

References

Andersson, M., Overby, L. J., & Sebestyén, S. (2009). Which news moves the euro area bond market?. *German economic review*, 10(1), 1-31.

Asplund, R. (2011). *The fundamentals that drive the Eurex European equity index and interest rate futures markets*. Technical report, Optima Investment Research, Inc.

Beirlant, J., Dierckx, G., Goegebeur, Y., & Matthys, G. (1999). Tail index estimation and an exponential regression model. *Extremes*, 2, 177–200.

Bessler, W., & Wolff, D. (2014). Hedging European government bond portfolios during the recent sovereign debt crisis. *Journal of International Financial Markets, Institutions and Money*, 33, 379-399.

Boothe, P., & Glassman, D. (1987). The statistical distribution of exchange rates: empirical evidence and economic implications. *Journal of international economics,* 22(3), 297-319.

Brys, G., Hubert, M., & Struyf, A. (2006). Robust measures of tail weight. *Computational statistics & data analysis*, 50(3), 733-759.

Candelon, B., & Straetmans, S. (2006). Testing for multiple regimes in the tail behavior of emerging currency returns. *Journal of International Money and Finance*, 25, 1187–1205.

Danielsson, J., & de Vries, C. G. (2000). Value-at-risk and extreme returns. *Annales d'Economie et de Statistique*, 239-270.

Danielsson, J., de Haan, L., Peng, L., & de Vries, C.G. (2001). Using a bootstrap method to choose the sample fraction in tail index estimation. *Journal of Multivariate Analysis*, 76, 226–248.

de Haan, L., Jansen, D. W., Koedijk, K., & de Vries, C. G. (1994). Safety first portfolio selection, extreme value theory and long run asset risks. In *Extreme value theory and applications* (pp. 471-487). Springer US.

Diebold, F. X., Schuermann, T., & Stroughair, J. D. (2000). Pitfalls and opportunities in the use of extreme value theory in risk management. *The Journal of Risk Finance*, 1(2), 30-35.

Drees, H., & Kaufmann, E. (1998). Selecting the optimal sample fraction in univariate extreme value estimation. *Stochastic Processes and their Applications,* 75(2), 149-172.

Duan, J. C. (1999). Conditionally fat-tailed distributions and the volatility smile in options. *Rotman School of Management, University of Toronto, Working Paper*.

Dumouchel, W.H. (1983). Estimating the stable index α in order to measure tail thickness: a critique. *Annals of Statistics*, 11, 1019–1031.

Embrechts, P., Resnick, S. I., & Samorodnitsky, G. (1999). Extreme value theory as a risk management tool. *North American Actuarial Journal*, 3(2), 30-41.

Fricke, C., & Menkhoff, L. (2011). Does the "Bund" dominate price discovery in Euro bond futures? Examining information shares. *Journal of Banking & Finance*, 35(5), 1057-1072.

Galbraith, J. W., & Zernov, S. (2004). Circuit breakers and the tail index of equity returns. *Journal of Financial Econometrics*, 2(1), 109-129.

Hartmann, P., Straetmans, S., de Vries, C.G. (2006). *Banking system stability: a cross-atlantic perspective*. In: Carey, M., Stulz, R. (Eds.), The Risks of Financial Institutions. The University of Chicago Press, Chicago and London, pp. 133–193.

Herlemont, D. (2005). *Risk Management Study for Managed Futures*. working paper, YATS Finance & Technologies.

Hill, B.M. (1975). A simple general approach to inference about the tail of a distribution. *The Annals of Statistics,* 3, 1163–1173.

Huisman, R., Koedijk, K. G., Kool, C. J. M., & Palm, F. (2001). Tail-index estimates in small samples. *Journal of Business & Economic Statistics,* 19(2), 208-216.

Huisman, R., Koedijk, K. G., & Pownall, R. A. (1998). VaR-x: fat tails in financial risk management. *Journal of Risk*, 1(1), 47-61.

Jondeau, E., Rockinger, M. (2003). Testing for differences in the tails of stock-market returns. *Journal of Empirical Finance,* 10, 559-581.

Jorion, P. (2009). Risk management lessons from the credit crisis. *European Financial Management*, 15(5), 923-933.

Koedijk, K.G., Schafgans, M.M.A., & de Vries, C.G. (1990). The tail index of exchange rate returns. *Journal of International Economics*, 29, 93–108.

Koedijk, K.G., Stork, P.A., & de Vries, C.G. (1992). Foreign exchange rate regime differences viewed from the tails. *Journal of International Money and Finance*, 11, 462–473.

Lux, T. (2001). The limiting extremal behaviour of speculative returns: an analysis of intra-daily data from the Frankfurt Stock Exchange. *Applied Financial Economics*, 11(3), 299-315.

Mandelbrot, B. B. (1963). The variation of certain speculative prices. *Journal of Business,* 36, 394-419.

Matthys, G., & Beirlant, J. (2000). Adaptive threshold selection in tail index estimation. *Extremes and Integrated Risk Management*, 37-49.

McNeil, A. J. (1998). Calculating quantile risk measures for financial return series using extreme value theory. *ETH Zürich, Departement Mathematik, Website „Alexander McNeil".* Available: http://www. math. ethz. ch/~ mcneil/pub_list. html.

McNeil, A. J. & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7, 271-300.

Mesnard, B., Margerit, A., Power, C., & Magnus, M. (2016). Non-performing loans in the Banking Union: stocktaking and challenges. *European Parliament Briefing*, March 18. Available: http://www.europarl.europa.eu/RegData/etudes/BRIE/2016/574400/IPOL_BRI(2016)574400_EN.pdf.

Müller, U. A., Dacorogna, M. M., & Pictet, O. V. (1998). Heavy tails in high-frequency financial data. *A practical guide to heavy tails: Statistical techniques and applications,* 55-78.

Odening, M., & Hinrichs, J. (2003). Using extreme value theory to estimate value-at-risk. *Agricultural Finance Review*, 63(1), 55-73.

Qi, Y. (2010). On the tail index of a heavy tailed distribution. *Annals of the Institute of Statistical Mathematics*, 62(2), 277-298.

Quintos, C., Fan, Z., & Phillips, P. (2001). Structural change tests in tail behaviour and the Asian crisis. *Review of Economic Studies*, 68, 633–663.

Schmid, F., & Trede, M. M. (2006). *Finanzmarktstatistik*. Springer-Verlag.

Schwaab, B., & Zhang, X. (2016). Tail risk in government bond markets and ECB unconventional policies (Working Paper). Available: http://www.berndschwaab.eu/papers/ScoreTailRisk.pdf

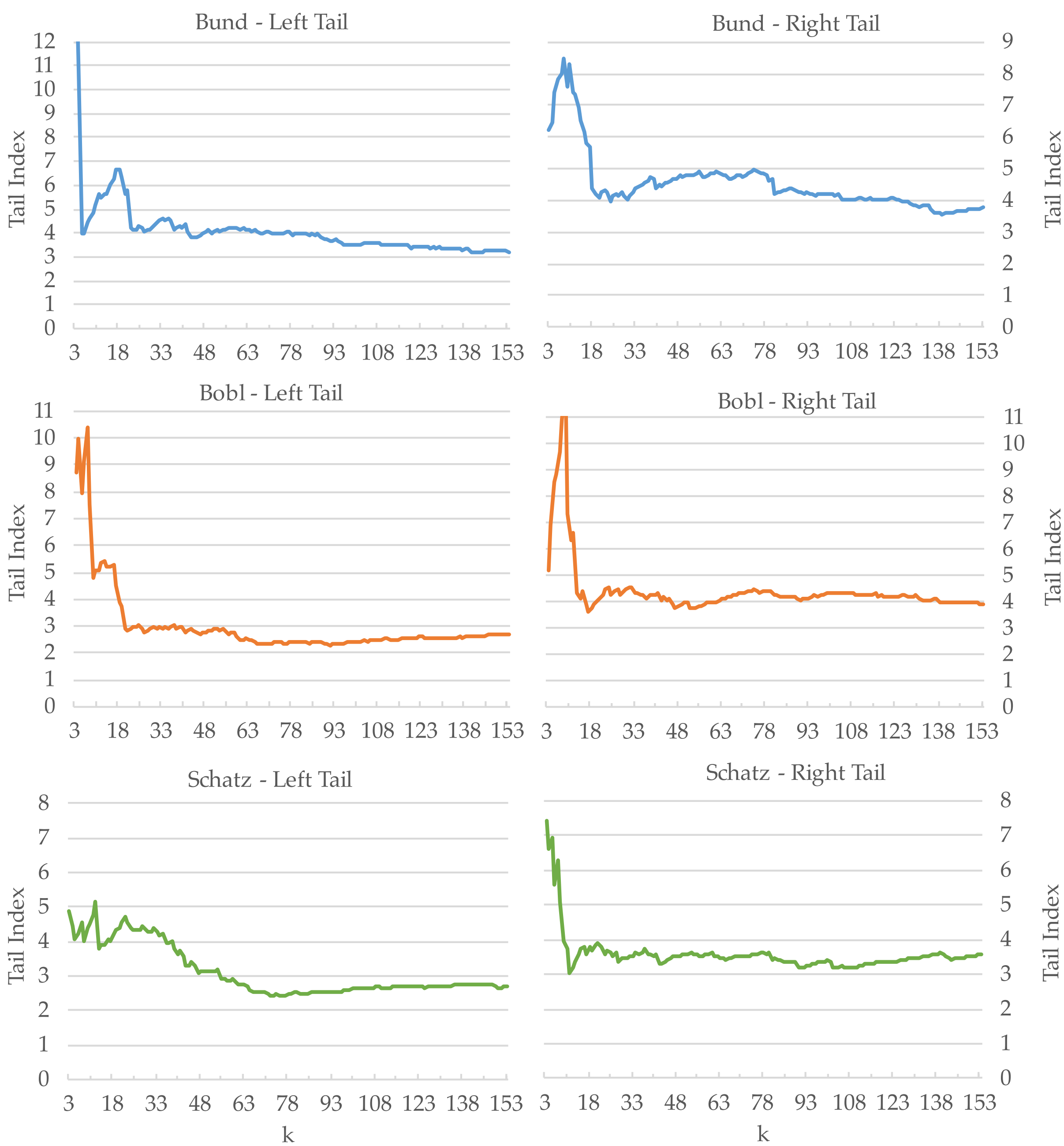
Straetmans, S., & Candelon, B. (2012). *Fats Tails in Small Samples*. Working Paper, Maastricht University.

Straetmans, S., & Candelon, B. (2013). Long-term asset tail risks in developed and emerging markets. *Journal of Banking & Finance*, 37, 1832-1844.

Straetmans, S., & Chaudhry, S. M. (2015). Tail risk and systemic risk of US and Eurozone financial institutions in the wake of the global financial crisis. *Journal of International Money and Finance*, 58, 191-223.

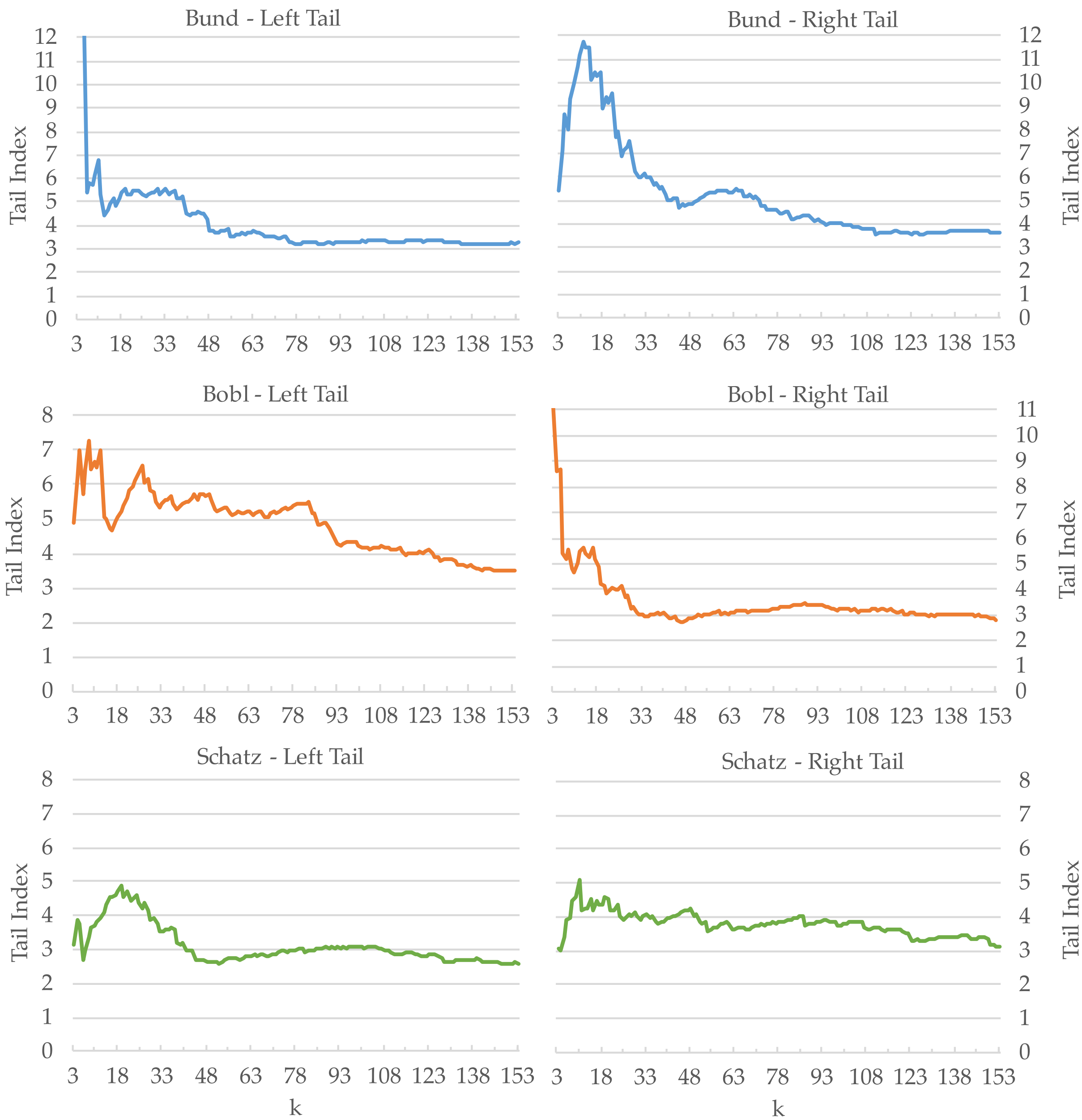
Werner, T., & Upper, C. (2002). *Time variation in the tail behaviour of Bund futures returns* (No. 2002, 25). Deutsche Bundesbank, Research Centre.

A Appendix



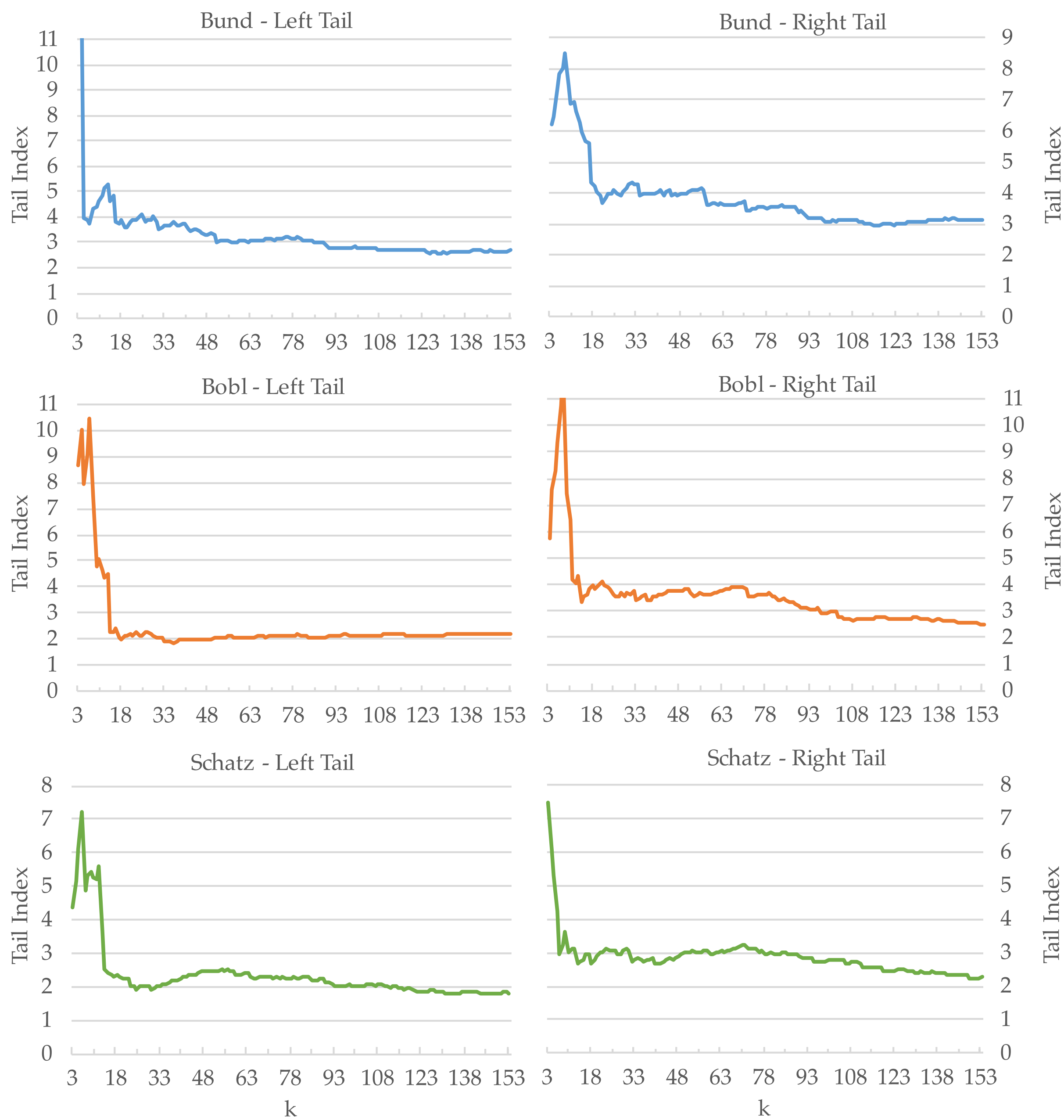
Notes: The graph shows Hill estimates for different numbers of extremes in the tail k, using the full sample period from 1999 - 2016 for the Bund, Bobl and Schatz future. The Hill estimates are calculated according to equation (3.2).

Figure A.1: Hill Plots - Full Sample



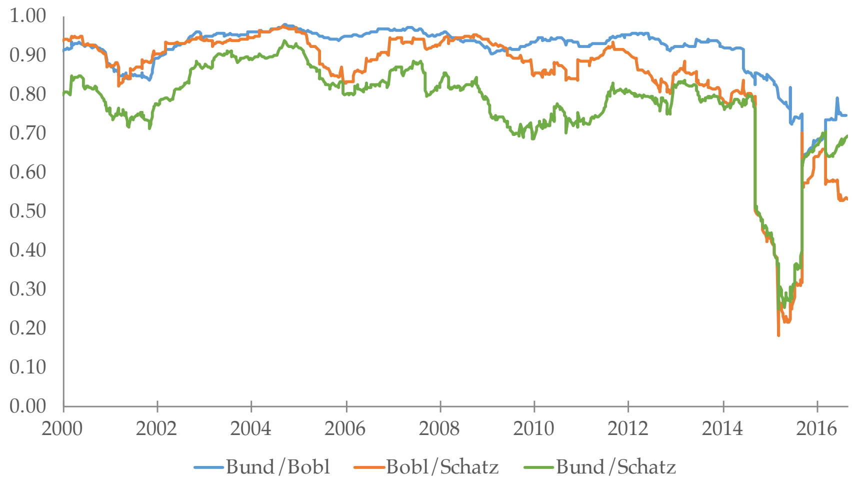
Notes: The graph shows Hill estimates for different numbers of extremes in the tail k, using a sample period from 01/1999 - 07/2008 for the Bund, Bobl and Schatz future. The Hill estimates are calculated according to equation (3.2).

Figure A.2: Hill Plots - First Sub-Sample



Notes: The graph shows Hill estimates for different numbers of extremes in the tail k, using a sample period from 08/2008 - 08/2016 for the Bund, Bobl and Schatz future. The Hill estimates are calculated according to equation (3.2).

Figure A.3: Hill Plots - Second Sub-Sample



Notes: The rolling yearly correlations are based on 250 past daily return observations of Bund, Bobl and Schatz future.

Figure A.4: Rolling Correlations of Daily Returns

	Bund Tail Index		Bobl Tail Index		Schatz Tail Index	
	Left Tail	Right Tail	Left Tail	Right Tail	Left Tail	Right Tail
01/1999 - 08/2016	3.94 (45)	4.28 (82)	3.07 (21)	3.84 (18)	3.35 (47)	3.39 (43)
01/1999 - 07/2008	4.82 (13)	6.21 (31)	3.99 (22)	4.32 (92)	2.73 (46)	3.63 (55)
08/2008 - 08/2016	3.48 (31)	3.69 (21)	4.68 (13)	3.37 (14)	3.90 (14)	2.80 (19)

Notes: The corresponding number of extremes k is shown in round brackets after each sub-sample tail index estimate. The values for $k$are based on the "Eye-Balling technique" with the Hill Plots in Figures A.1, A.2 and A.3.

Table A.1: Tail Indices based on Hill Plots

	Bund Tail Index		Bobl Tail Index		Schatz Tail Index	
	Left Tail	Right Tail	Left Tail	Right Tail	Left Tail	Right Tail
01/1999 - 08/2016	4.52	4.87	3.61	4.59	3.28	4.37
01/1999 - 07/2008	4.76	6.10	3.20	5.87	2.83	4.70
08/2008 - 08/2016	4.07	4.52	4.21	4.34	3.49	3.42

Notes: The presented tail index $\hat{\alpha}$ values are estimated through the method by Huisman et al. (2001).

Table A.2: Alternative Tail Index Estimates